4.10.2 Trick number 2. The cylindrical test object can be rolled 180° in the fixture (vee block) and remeasured. This establishes the centerline of the vee block.

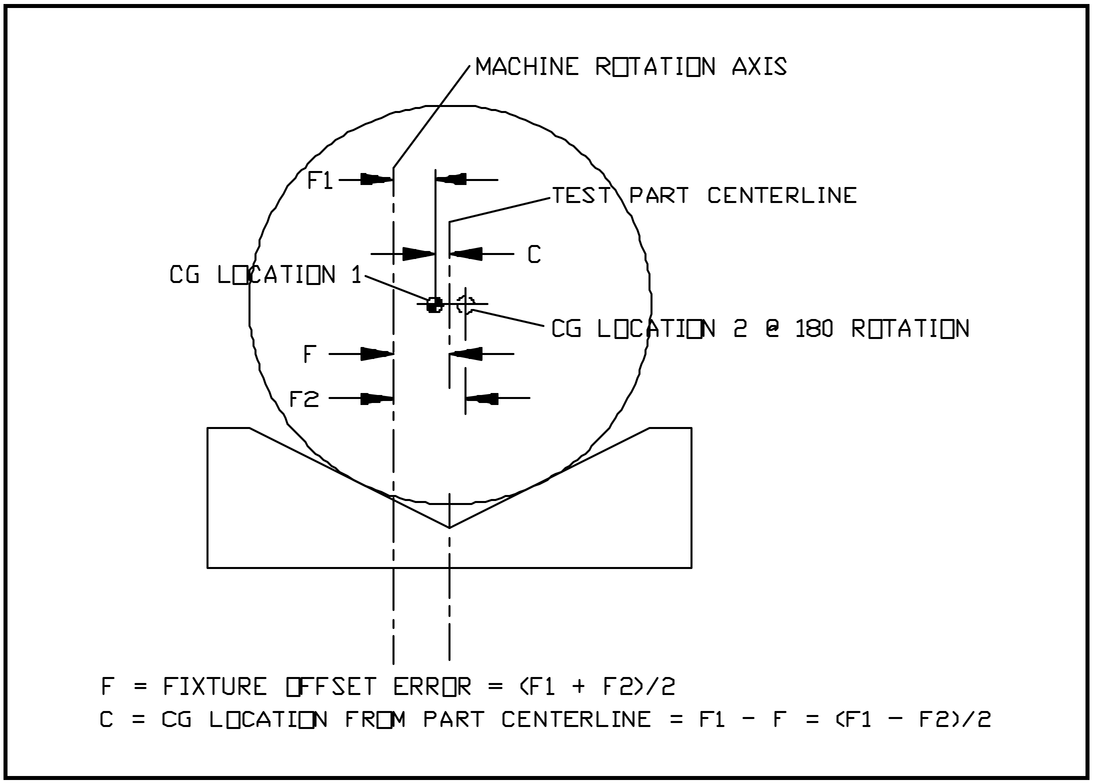


Figure 8-Work Reversal Method Eliminates Fixture Error

The process here is similar to the end for end case. If the vee block were centered on the measurement axis of the instrument, rolling the object 180° will change the sign of the CG offset, but the magnitude will remain the same. If the magnitudes differ, then this indicates that the vee block is not exactly centered. You then have two choices: if there is a large difference, then you can reposition the vee; if the difference is small, then computing half the difference between the two readings gives the true CG offset. As in the previous case, the exact position of the adaptor no longer has any effect on the answer. If several test objects must be measured, the fixture offset may be calculated and used as a correction for the measurements.

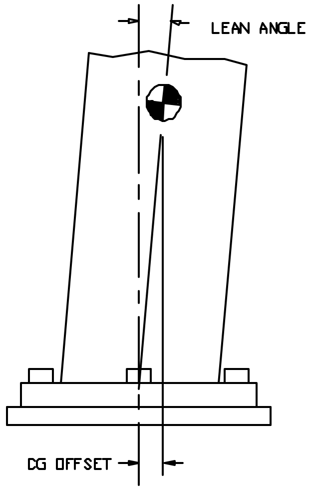
Note: Tricks 1 and 2 are called the "work reversal method".

Note: This method eliminates fixture position error, but does not eliminate errors shown below due to poor fit in the fixture.

NOTE:. It is very important to note that these methods only work if the cylinder is round and square, and the vee block contacts the cylinder on precision surfaces. Otherwise, you have no way of separating fixture misalignment from machining errors in the fixture.

4.11 Effect of fixturing error

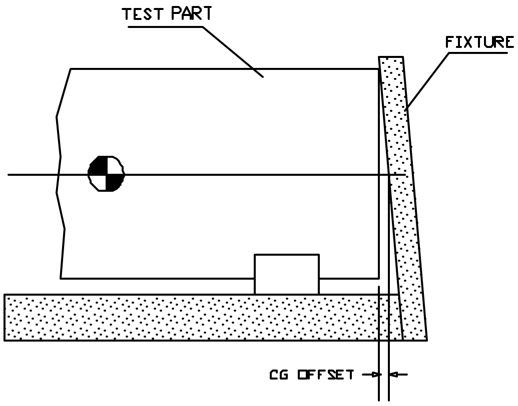
4.11.1 Effect of fixturing error on mass properties measurements Dimensional inaccuracy in fixtures adds directly to the inaccuracy of mass properties measurements. For example, if fixture error causes the test part to lean slightly to one side, then the instrument will indicate an apparent CG offset. Rotating the test table 360 degrees will not detect this lean error, since it causes the test object CG to go through a maximum and minimum reading in a manner similar to that which occurs for true CG offset. This same lean for a tall slender object will cause a horizontal shift in mass at the upper end of the object causing an inaccurately large MOI to be measured. The POI measurement will also be adversely affected. Fixturing is generally more critical for CG and POI measurements than MOI, as discussed below.



OUT OF SQUARE TEST PART INTERFACE
MAY CAUSE TRUE CG OFFSET

OUT OF SQUARE FIXTURE MUST BE
SHIMMED TO ELIMINATE LEAN ERROR

4.11.2 Effect of fixture error on CG Fixturing errors in the horizontal plane have a direct one to one relationship to CG error. A 0.01 inch fixturing error translates to a 0.01 inch measurement error. The goal for fixturing accuracy should be about one fifth to one eighth of the allowed CG tolerance. For example, if object CG must be within +/-0.008 inch of the centerline of the object, then to meet the one eighth goal, the interface surface of the object must be round and concentric with the centerline within +/- 0.001 inch.

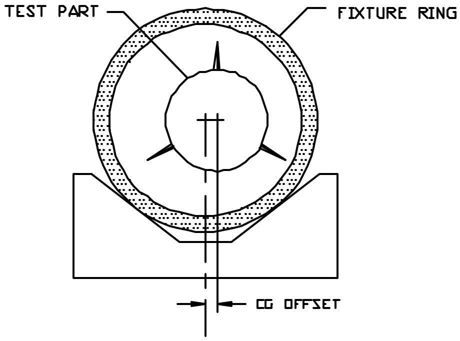


OUT OF SQUARE REFERENCE SURFACE
CAUSES CG OFFSET ERROR

The interface must also be perpendicular to the centerline to a tolerance that insures the part does not lean more than 0.001 inch at the CG height, causing an erroneous CG offset. The required tolerance for lean can be calculated from the formula:

$$TIR = DX/8H$$

where TIR = total indicator vertical runout at diameter D of the object/table interface
X = CG offset tolerance of object
H = CG height of the object



FIXTURE RING CONCENTRICITY ERROR
CAUSES CG OFFSET

This formula results in a perpendicularity tolerance which is one eighth the allowed CG offset, X. The tolerance on perpendicularity is surprisingly tight for most tests. For a rocket with a 100 inch CG height, an interface diameter of 10 inches, and a CG offset tolerance, X, of +/- 0.004 inch, the required perpendicularity (to meet the 1/8 ratio) would be 0.000,050 inch TIR!

Clearly, this is impossible. One solution is to mount the object on a tilt/translation fixture and center the object *at the CG height*. A more practical and cost effective solution is to fixture the object horizontally to avoid this difficult perpendicularity requirement.

4.11.3 POI: The magnitude of POI error is proportional to the difference between the axial and transverse moments of inertia of the test item (if these are equal, then alignment is not critical; if they are very different, such as would occur with a long thin shaft, then alignment is the limiting factor in measurement accuracy). Never measure POI without first dial indicating the object at two different heights to make certain the object is centered and does not lean. Do not rely on the fit in a fixture to establish position. For a slender object, a runout at the top of the object of 0.005 inch TIR may be enough tilt to cause the object to fail the POI specification. If you then add correction weights to compensate for this, you will be creating an unbalance!

MOI: Fixturing accuracy is not critical except if the payload is tall and thin. The reason for this is that the error is proportional to the square of the ratio of radius of gyration ("k") and fixture offset error ("d"). Generally the fixturing error is less than 1% of the radius of gyration, so the resulting error will be less than 0.01%. This relationship is derived below, using the well-known formula for translations of axes.

$$I = Mk^2 + Md^2 = Mk^2 \left(1 + \frac{d^2}{k^2}\right)$$

$$\mathit{If}\ \frac{d}{k} < \frac{1}{100}\ ,\ \mathit{then}\ \frac{d^2}{k^2} < \frac{1}{10{,}000}$$

4.11.4 Using a Precision Dummy Payload One very convincing method to verify fixture accuracy is to construct a precision test weight with known mass properties which interfaces with the fixture in the same way as the real payload. For example, this weight might be a simple cylinder of constant diameter. If the mass of a solid cylinder would be too large, but you need a large diameter to interface with the fixture, you can use a small diameter solid cylinder with a larger diameter disc attached to each end.

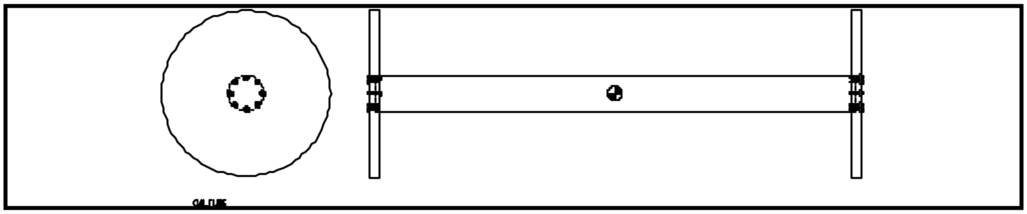


Figure 10 - Solid metal calibration weight simulates mass and diameter of low density rocket.

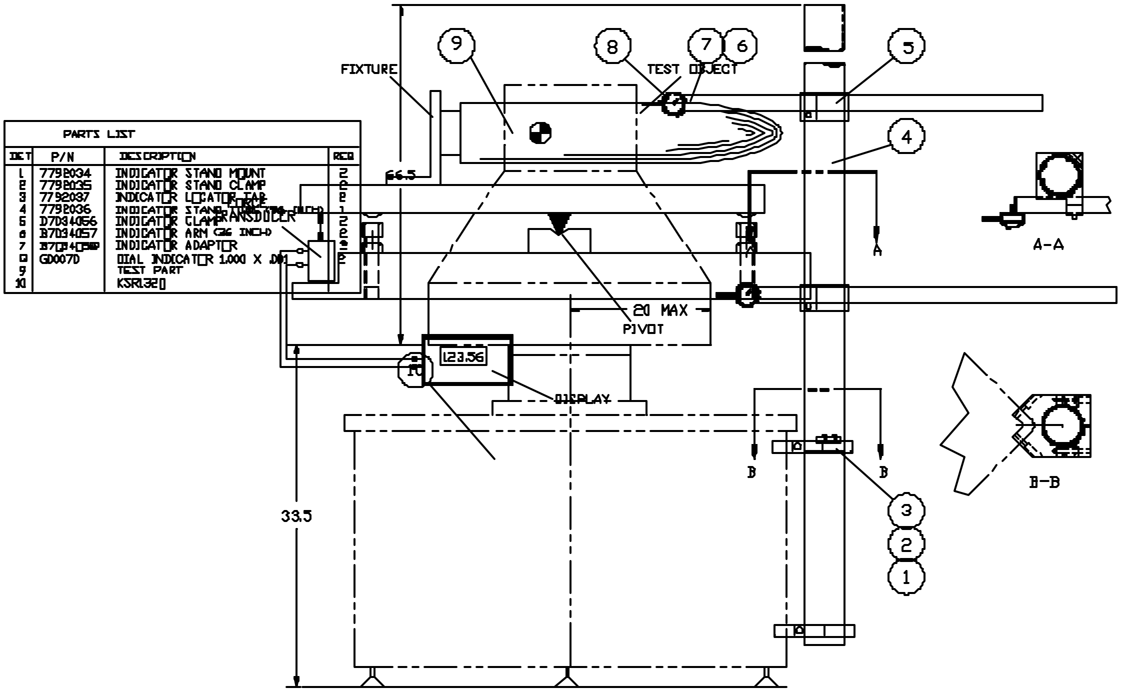


Figure 4 - A good way of fixturing an object for POI measurement is to dial indicate the object and adjust its position.

5.0 Methods used to measure CG Location

There are three basic static methods used to measure the CG of an object and 2 dynamic. Static methods depend only on the force of gravity acting through the test object CG and are preferred over dynamic methods.

In contrast to static methods, the dynamic methods require spinning the object or oscillating it to measure MOI in several positions. Consequently, dynamic methods are generally less accurate and more difficult to accomplish than static methods.

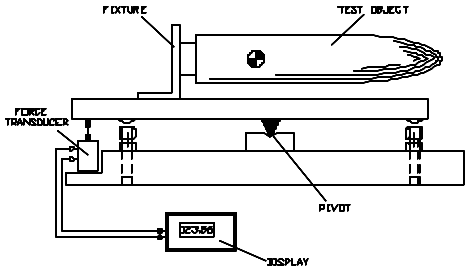
5.1 Static Methods

Unbalance Moment Method This method uses a pivot axis which supports most of the weight of the test object. The table CG machine senses the overturning moment produced by a displacement of the test object CG from the pivot center of the table.

Multiple Point Weighing Method --(also called "3-Point Weight and CG Instrument") or "Reaction Method") The CG of an aircraft is traditionally determined by placing scales or load cell platforms under the three wheels of the aircraft and calculating the CG location from the difference in force measurement at these three points. An instrument can be constructed on this same principle, wherein a test platform is supported by three or more load cells.

Mechanical Repositioning Method This method uses a pivot axis which supports <u>all</u> of the weight of the test object. To measure CG, the object is moved so that a balance is achieved about the pivot point.

5.2 Description of Unbalance Moment Method This method uses a pivot axis which supports most of the weight of the test object. The pivot is shown as a knife edge in the following illustration. However, modern instruments use a spherical air bearing as the pivot. A force transducer senses the overturning moment produced by a displacement of the test object CG from the pivot center of the table. Measuring this moment and dividing by the test object weight will yield the CG displacement from the center.



5.2.1 Instrument pivot type - Pivot friction affects the sensitivity of the instrument. The best instruments use a gas bearing. Crossed-web flexures are equally low in friction, but they have a bending moment which can result in a non-linearity for soft transducer machines. Knife edges have moderate sensitivity when new, but they rapidly deteriorate as the edge is worn down and are easily damaged. Roller or ball bearings have relatively high friction which seriously limits accuracy. These bearings can also be damaged by impact.

Air Bearing Pivots Air bearings consist of a precision rotor and a precision stator separated by an air gap that is less than 0.0005 inches thick. Air is introduced to the gap through jewel orifices that meter the air and provide dynamic centering of the bearing. Machining accuracy on these bearings is better than 30 millionths of an inch. This is what makes air bearings so expensive and difficult to make.

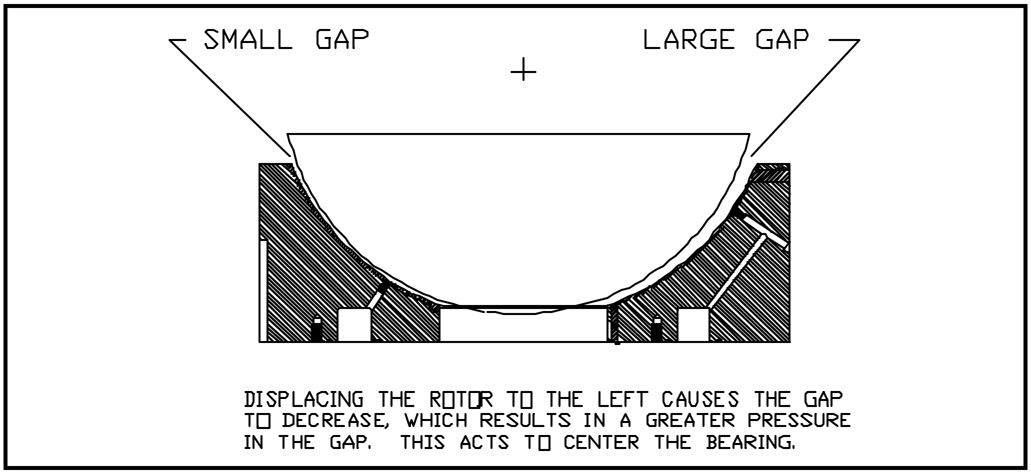


Figure 7 - Partially choked flow through small orifices produces dynamic centering of air bearing.

Dynamic centering action Contrary to intuition, air bearings have greater stiffness and precision than any other type of bearing.

The reason is that an air bearing is a dynamic device. If air is supplied through a single opening to the gap between the ball and the cup of a spherical air bearing, then the bearing would only operate successfully if the external forces were exactly in the center of the upper plate. A side load would cause the rotor of the bearing to move sideways so that one edge rubbed against the stator. Increasing the amount of air pressure in the plenum of the bearing would not improve the situation, since the additional available air would flow out the side which had the larger gap.

Air bearings made by Space Electronics minimize this effect by using independently supplied segments and small diameter jewel orifices which operate in a partially choked condition.

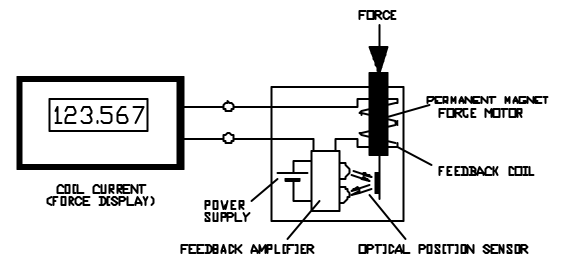
Under conventional operation with the payload centered on the interface table, the amount of air flow through each orifice is such that a pressure drop of approximately one half the pressure in the plenum occurs.

A minute movement of the rotor of the bearing results in a restriction of the flow on the side of the bearing which has the smaller gap and an increase in flow on the opposite side of the bearing. This produces a self-compensating or centering action of the bearing, since a reduction of the air flow on the low side increases the pressure in the gap on that side and an increase in flow on the high side reduces the pressure on that side. Proper selection of orifice sizes and cavity configurations permits the bearing to remain centered within about .0001 when subject to side loading.

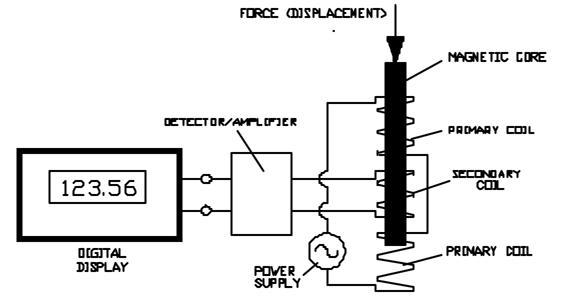
5.2.2 Instrument Transducer type - The best instruments use an active force restoration transducer (introduced to the CG measurement industry in 1988 by Space Electronics) to measure moment.. This type of transducer can be built with a dynamic range of 300,000 to 1, a linearity of 0.001%, is very stiff (so that lean error is minimized), and has excellent overload protection.

Load cells can be used in place of the active rebalance transducer. These have high stiffness, but overload protection is very poor, so that the instrument has to be protected by air cylinder lockouts or other means while the test object is being loaded. Dynamic range of the load cell is limited to about 2000 to 1.

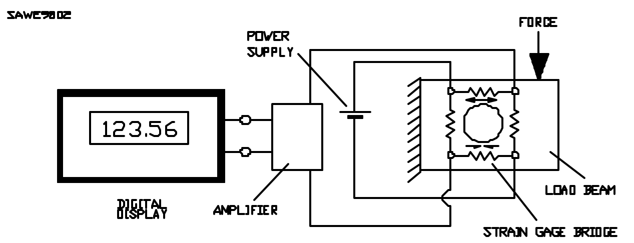
A third transducer type is a torsion rod/LVDT system which measures the lean angle that results from the unbalance moment. These transducers have good overload protection, but they are very soft, so that lean error is large for tall test objects and considerable time is required for the system to stabilize after the test object is installed or rotated. Linearity and dynamic range are slightly better than load cells.



ACTIVE FORCE REBALANCE TRANSDUCER
(HIGHEST ACCURACY & DYNAMIC RANGE)



LVDT TRANSDUCER
(LINEAR VARIABLE DIFFERENTIAL TRANSFORMER)
(GOOD OVERLOAD PROTECTION, MODERATE ACCURACY)



STRAIN GAGE LOAD CELL TRANSDUCER
(EASILY DAMAGED BY OVERLOAD, LIMITED ACCURACY)

Instruments can also be made which contain no electrical transducer. These instruments use sliding weights or weights which are driven by a lead screw. These instruments are sensitive and linear; their disadvantage is that there is no electrical output to a computer or printer, and skill and time are required to make a measurement.

5.2.3 Rotary table CG instrument Adding a rotary table to the CG instrument greatly improves measurement accuracy. Most systematic errors are automatically eliminated or can be eliminated in machine setup. The table rotates the test article to four locations (0, 90, 180, & 270 degrees) where static CG measurements are made.

> **!** The axis of measurement becomes the center of rotation of the table. This eliminates the need to accurately determine the relationship between the instrument pivot axis and the mounting surface of the instrument.

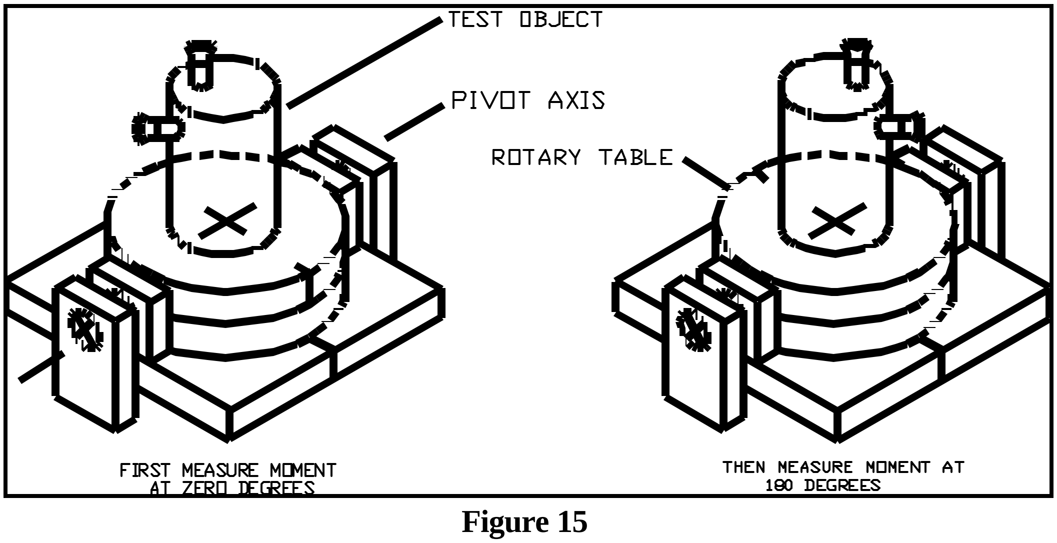
! For cylindrical test parts, or parts that can be accurately located in a fixture with a cylindrical reference surface at the nominal CG location, a dial indicator may be used to bring the part or fixture centerline concentric with the center of rotation of the instrument to within extremely close tolerances. This eliminates zero reference offset errors.

! For tall cylindrical parts, two dial indicator readings may be made: one close to the table and another at a location well above the table. A tilt table, shimming, or other means of adjustment, will allow the operator eliminate errors due to the part axis leaning away from the machine rotational axis.

! Another common system error occurs from improper leveling of the machine. This causes lean, which may be interpreted as CG offset. With the rotary table machine, this error is eliminated by taking readings for each axis which are 180 degrees apart. The lean error is equal for both measurements and is therefore subtracted from the result.

! Taking two readings for each axis also eliminates other systematic errors such as transducer zero offsets.

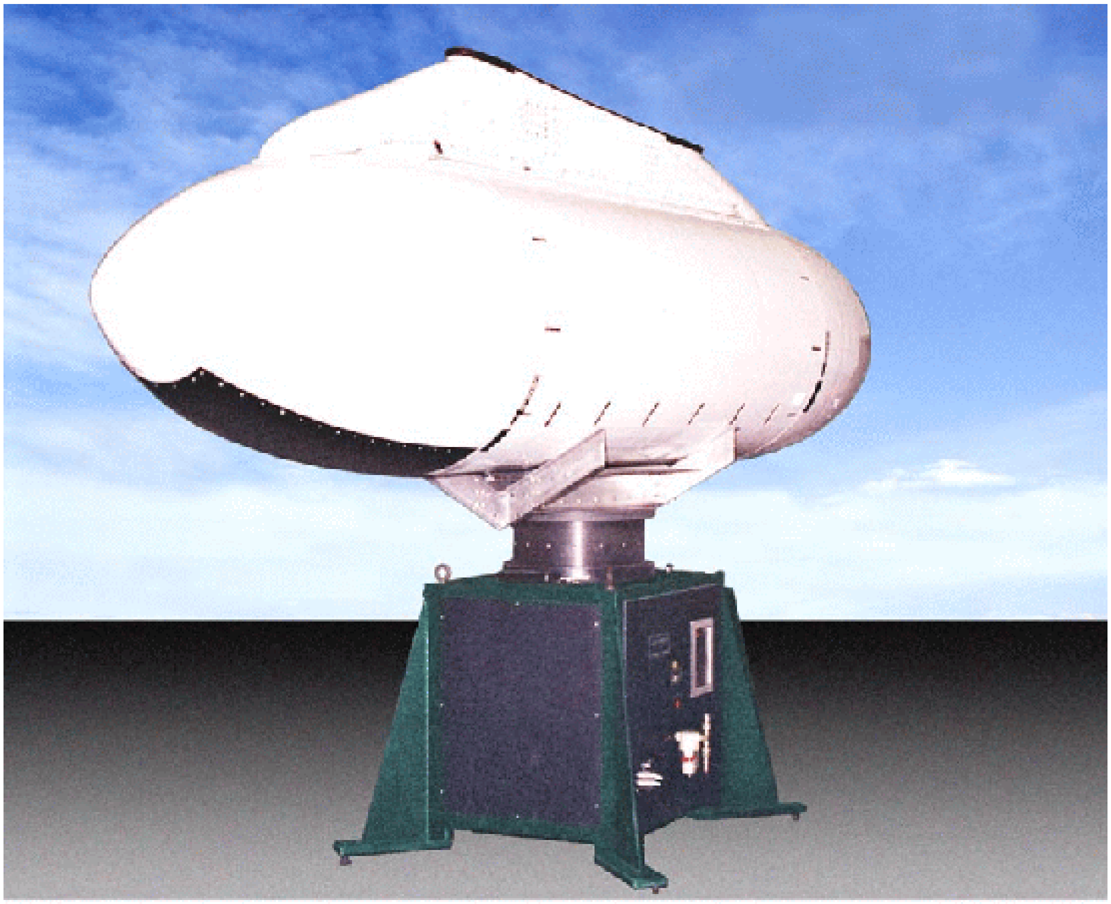
5.2.4 Spherical gas bearing rotary table instrument When the rotary table concept is implemented with a spherical gas bearing, the resulting instrument is capable of higher accuracy than any other method. The bearing acts as both a pivot and a rotary table. The use of a gas bearing makes it easy to design an instrument which can measure both CG and Moment of Inertia. For maximum accuracy, the overturning moment produced by a displacement of the test object CG from the center of rotation of the table is sensed using a force restoration transducer. Measuring this moment and dividing by the test object weight will yield the CG displacement from the center. This type of instrument does not measure weight so a separate scale must be used. The weight data can be automatically acquired by the mass properties machine software and used to calculate CG location.



FIRST MEASURE MOMENT AT ZERO DEGREES

THEN MEASURE MOMENT AT 180 DEGREES

Figure 15



The Object in the photo is supported on spherical air bearing. Inverted torsion pendulum measures moment of inertia and force restoration transducer measures two-axis center of gravity location.

Photo shows Space Electronics Model KSR Mass Properties Instrument with object mounted in vee fixture. Both center of gravity and moment of inertia are measured in a single setup. Instrument CG measurement uncertainty is +/- 0.001 inch and moment of inertia measurement uncertainty is 0.1%.

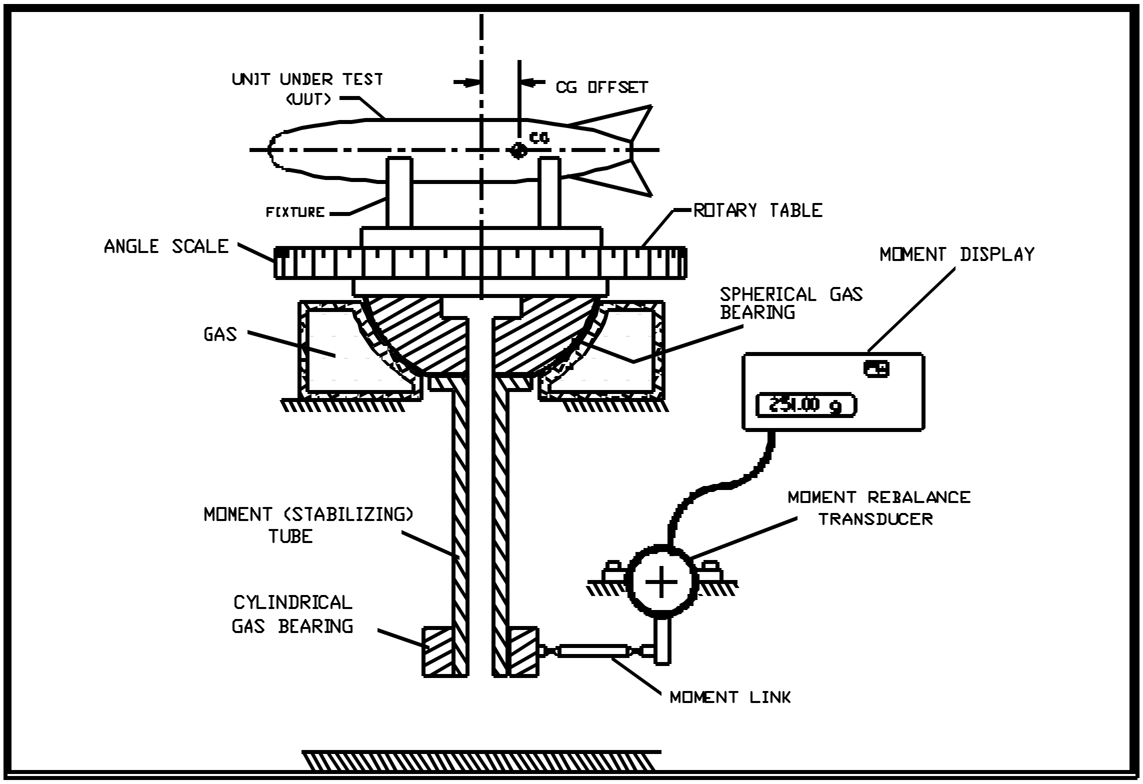


Figure 16 Basic elements of the Space Electronics Model KSR instrument. The spherical gas bearing creates both a precision rotary table and a frictionless pivot. Active force rebalance transducer measures overturning moment due to CG offset from center of rotation.

SUMMARY OF BENEFITS & SHORTCOMINGS

Benefits:

1. Accuracy is higher for this type of instrument than any other method.
2. This instrument is easily configured to measure MOI as well as CG moment.
3. Fixturing error is minimized since the rotary table allows cylindrical parts to be dial indicated. 4. Levelling error is eliminated by using the rotary table to take data readings at test part locations which are separated by 180 degrees.

Shortcomings:

1. A separate weight platform must be used to determine test part weight.
2. It is a more expensive system for CG measurement than the 3-point method. (Cost is less than the spin balance method).
3. It is slower than the 3-point method which may make it less suitable for high volume use.

5.3 Multiple Point Weighing Method -- A test platform is supported by three or more load cells, and the CG location is calculated from the difference in force measurement at these three points. In the past, the accuracy of this method has been limited by the dynamic range of load cells, so that these instruments were not suitable for projectile and missile measurements. The introduction of force rebalance technology to CG measurement by Space Electronics in 1988 has reduced force measurement errors by a factor of 30. When this technology is applied to the Multiple Point Weighing Method, accuracy improvement is great enough so that this method now becomes acceptable for many applications. This instrument measures weight as well as CG.

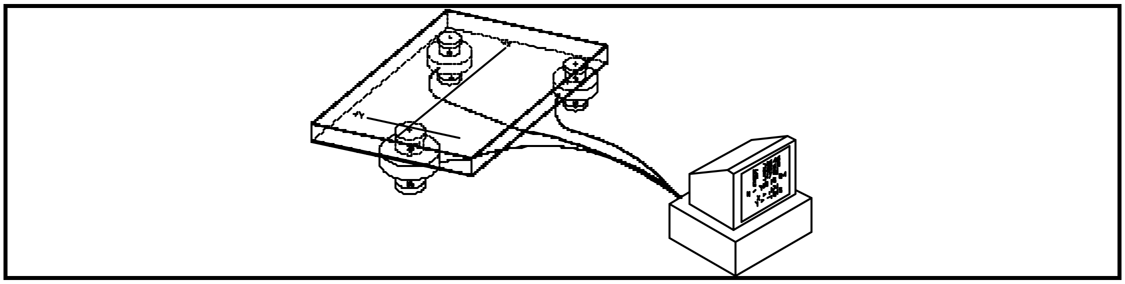


Figure 17 Multiple Point Weighing CG Machine - Fast and easy to use - moderate accuracy.

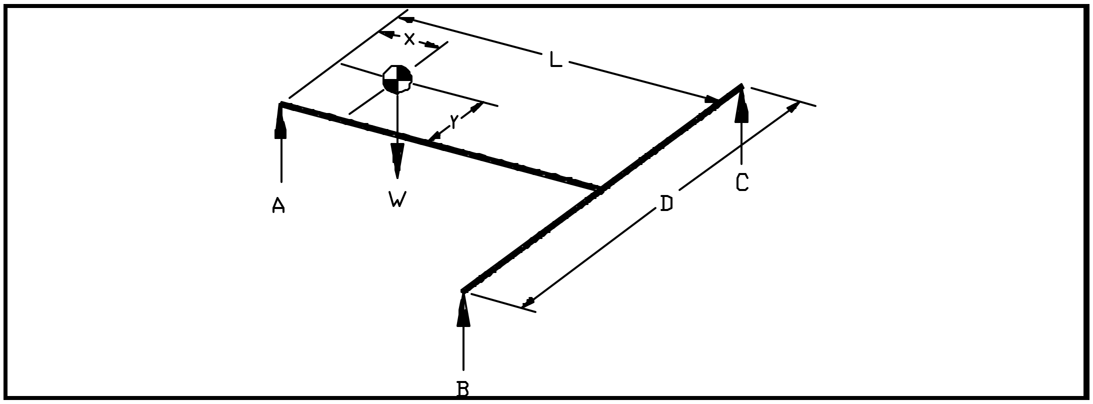


Figure 18

5.3.1 Calculating weight and CG location

To determine part weight (W) and CG coordinates X and Y, three force transducers are typically used to support a frame which in turn supports the object.

$$\boxed{W = A + B + C}$$

where A, B, and C are force readings on the three force transducers.

To determine CG, take moments about A, where X and Y are the CG measurement coordinates.
If all the transducers outputs are set to zero when fixturing is in place, the equations above are used to determine the CG location of the test part. In practice, tare readings are subtracted from the part measurements and the values above represent the <u>net</u> A, B, and C forces required to support the part weight and CG offset moment.

$$\Sigma M_x = (B+C)L - WX = 0$$

$$\Sigma M_y = \frac{CD}{2} - \frac{BD}{2} - WY = 0 = \frac{D}{2}(C-B) - WY$$

$$X = \frac{(B+C)L}{W}$$

$$Y = \frac{(C-B)D}{2W}$$

SUMMARY OF MAJOR BENEFITS AND SHORTCOMINGS

Benefits

1. Measures both CG and weight.
2. By using the latest force rebalance transducers and optimum geometry, sensitivity is adequate for most applications
3. For a given CG offset moment capacity and part weight, it is the lowest cost automatic system.
4. It is most suitable for very heavy parts with tight CG location tolerances
5. This is the fastest CG measurement method. Total time to make a measurement of 2 axis CG is less than 30 seconds.

Shortcomings

1. A separate instrument must be used to measure MOI if this quantity is required.
2. It is highly sensitive to and not readily correctable for lean error caused by leveling.
3. The machine axis zero point is difficult to define. Unlike rotary table machines, the object cannot be dial indicated. Fixturing errors may be relatively large.

5.4 Repositioning Method of Cg Measurement A third method of CG measurement is the free-pivot method where the test object is balanced on a pivot and allowed to tilt. The test object is moved relative to the pivot of the instrument until a balance is obtained. Some means is then required to measure the final position of the object This method of CG measurement is:

Inexpensive
Very time-consuming
Generally the least accurate of all methods

The first two techniques discussed do not require an “instrument”. CG is measured using common objects that are laying around the lab. Both of these methods have a fundamental flaw: there is no accurate method to transfer the answer to the coordinate system of the test object.

5.4.1 Hanging pivot If an object is suspended from a knife-edge, the CG of the object will lie directly below the center of pivot of the knife-edge. The position of the test object is moved relative to this pivot point until a balance condition is achieved. Torpedos and projectiles can sometime be measured using this method.

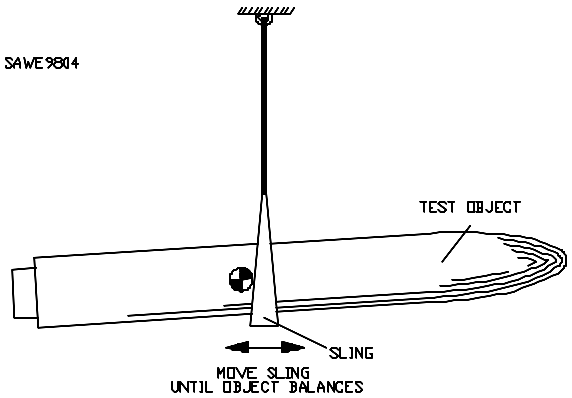


Figure 19

Hanging systems such as this one where the test object CG is always below the pivot point have the disadvantage that their sensitivity is low. Shifting the lateral CG of the test object results in only a small change in the level condition of the instrument. The amount of tilt which results from a CG offset is a function of the CG <u>height</u> difference between the object and the knife edge. Reducing this distance increases sensitivity. However, it also increases measurement time, since the object rocks back and forth at a slower rate when sensitivity is increased. Each time the item is moved, the operator must wait for the system to settle again. In this respect, this method is like using the old beam balance scales. For very sensitive systems, it can take hours to make a single measurement. (In contrast, force rebalance techniques are up to 100 times more accurate but only require seconds for a single measurement).

The sensitivity of the instrument is directly dependent on the accuracy of the means used to detect the level of the instrument. Bubble levels are available with the sensitivities as great as one division for an angle change of 0.00005" per ft. A more practical figure to use in determining level accuracy, however, is approximately 0.002" per ft., since the position of the bubble level relative to the measurement axis of the instrument is difficult to adjust to a closer tolerance than this. Even the smallest error in the position of the bubble level relative to the structure of the instrument results in a large error in measured CG location.

This method is extraordinarily tedious to use. The object must be lifted and repositioned 10 or 20 or 30 times, depending on the accuracy required. Each time the swinging must be allowed to damp out and the level condition read. Since there is no readout to indicate how far to move the object, the process is strictly trial and error. The initial cost of this method is the lowest of any type of CG measuring technique, but often the labor required to make a measurement more than offsets the cost, causing this method to be in fact, the most expensive of all techniques.

Once you have obtained an accurate balance, how do you relate the final position of the object to the pivot axis of the knife edge? Unless you can do this accurately, you have accomplished nothing by tediously balancing the object. One method is to use a transit. You center the crosshairs on the knife edge pivot and then swing down to view the object. The object is marked, and then removed from the CG fixture and placed on a coordinate measuring machine, where the location of the mark is measured. Generally the accuracy of this method is about ±0.040 inch.

5.4.2 Measuring CG using a "broom handle" The simplest method of measuring CG is to balance the object on a round rod. This method only works if :

a) the object has a low profile
b) the object has a rigid surface that will not be indented if its entire weight is supported on a narrow rod
c) the surface of the object is flat and smooth.

Since the total CG of the test object lies above the pivot axis, the object can tip in either direction when the pivot point is near its CG. This deadband results in an error in CG location. The magnitude of this error is proportional to the amount of angular tilt of the test object which is permitted. Reducing this tilt will decrease the magnitude of this error. However, the small amount of travel makes the instrument extremely tedious to operate - there is no advance warning that the balance point is being reached and extremely fine adjustments of the test part position must be added to prevent overshooting this very narrow point.

To get a better feeling for the problems in using this method, lay a ruler at a right angle on top of a round pen. You will notice that the ruler can never be made to balance on top of the pen. The reason is that the CG of the ruler is above the pivot point (contact point between objects), so that the ruler has two states of equilibrium. The trick to using this method is to roll the pen in one direction until the ruler flops over, and mark the contact point with the pivot; then roll the pen in the other direction and mark where it flops back. The CG of the ruler is near the center of these two points. (It is not exactly at the center, because of pivot friction, and the error introduced by inequalities of tilt angle when the object is at rest at its two equilibrium positions). This method works pretty well for a ruler, because it is long and thin. The length allows you to limit the amount of tilt, and the low CG height limits the deadband.

Now try using this method to measure the CG of a coffee cup. You will notice that the distance between the two CG marks increases, since the CG is higher and moves a greater lateral distance when the cup moves from one equilibrium state to another. Furthermore, it is hard to limit the tilt so that it is equal in both directions.

One catch to this method is that there is no way to accurately transfer the answer to the coordinate system of the object. You can see when the object tilts, but how do you relate this location to the CG along the length of the object?

Note: Cylindrical objects cannot be tested using this method, since this would result in the intersection of two rounded surfaces, causing the entire weight of the object to be supported on a single point. In most instances, the object would be damaged. Furthermore, the object will tend to rotate about a vertical axis, so that its orientation relative to the pivot axis will be altered.

5.4.3 CG Measuring structures which make use of repositioning The concepts discussed above can be incorporated into an instrument with a mounting surface and an accurate relationship between pivot and center pilot on the mounting surface. These instruments require the object to be moved. Therefore, they represent obsolete technology and are mentioned only to make this summary historically correct.

Figure 20 illustrates the same type of instrument in an unstable condition. There is no vertical counterweight so that the CG of the moving system is above the pivot point for any test object. Rotation stops are provided to prevent excessive motion of the test object. This unstable condition results in a hysteresis or deadband that limits the accuracy of the instrument. Decreasing the gap between the stops reduces the deadband, increasing sensitivity. It is impractical to reduce this gap more than a certain amount. The ultimate accuracy with this type of instrument can be obtained by leveling the instrument with one stop in contact, and then moving the object until the platform tips. This technique has, in effect, reduced the deadband to zero.

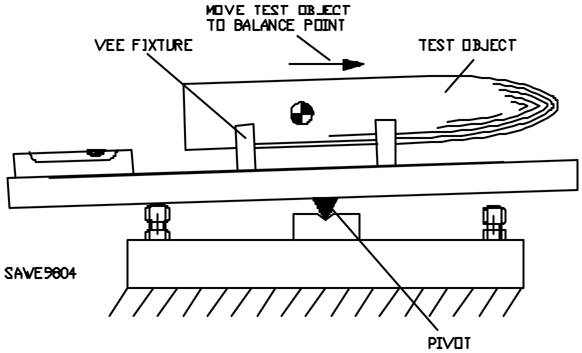


Figure 20

Figure 21 illustrates a stable type of free pivot instrument. In this instrument, the lower counterweight is adjusted vertically until the total system CG is slightly below the pivot point.

The instrument will then exhibit maximum sensitivity but will be stable. This instrument will only work for an object which has a straight surface and can be slid sideways in the fixture.

One type of re-positioning CG instrument which we used to manufacture in the 1970's consisted of a test platform which is mechanically coupled to a counterweight. A motion of this test platform results in an equal and opposite motion of the counterweight maintaining the balance of the structure of the instrument when the test part is re-positioned.

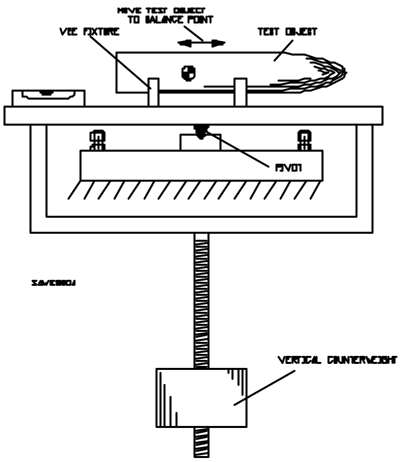


Figure 21

All free balance CG measuring systems have the same disadvantages. The measurement is extemely tedious to perform, since re-positioning of the object often requires that it be lifted temporarily. After the object is moved, the new position of the object must be determined in order to identify the location of the object CG. This is the major accuracy limitation of this method. In contract, the object is fixture at a precisely known location when using moment measuring instruments. Repositioning instruments often require a relatively elaborate fixture which is usable for only one particular type of test part.. The sensitivity of the techniques which pivot the test part above or below its CG is very poor, but the sensitivity of the methods which approximately align the pivot axis with the CG is high. However, these high sensitivity systems require the use of vertical counterweights; errors in the alignment of these counterweights with the pivot axis of the instrument can cause large measurement errors unless the structure is re-balanced after each height adjustment is made. Unstable free balance systems have the additional disadvantage that the test operator has no way of knowing when he is approaching a balance condition.

SUMMARY OF BENEFITS AND SHORTCOMINGS

Benefits

1. Lowest cost method.
2. Can achieve high sensitivity (but not necessarily high accuracy).
3. Inherently safe in explosive environments.

Shortcomings

1. Accuracy is generally limited because of difficulty of determining location of object
2. Tedious and time consuming to operate.
3. Can’t be used for irregularly shaped objects.
4. Accuracy depends on skill of operator.

5.5 Dynamic methods of measuring CG

In contrast to the methods described above, these methods require that the test object must be moved during measurement. The data obtained during these measurements is related to product of inertia as well as CG; some method must be used to eliminate the effect of product of inertia in order to derive CG location.

Spin Balance Method -- The test object is rotated and force transducers sense the reactions on the bearings which support the part during rotation. These forces are due to both gravity and centrifugal force (the higher the spin speed, the less significant the gravity force is). The CG location of the part may then be separated from the dynamic unbalance of the part using calculations that involve the magnitude of the bearing forces and their phase relationship. This method was used extensively before the 1970's, when force measuring technology was rather crude. However, since the development of single-point load cells with solid state amplification there is no longer any justification for using this method.

Moment of Inertia Method -- The test object is mounted on an inverted torsion pendulum (moment of inertia instrument) and successive moment of inertia measurements made for at least three positions of the test object. The CG location can then be calculated from the small change in MOI which results from moving the principal axis of the object.

These methods are described in more detail in the following sections..

5.5.1 Spin Balance Method of Measuring CG -- The test object is rotated and force transducers sense the reactions on the bearings which support the part during rotation. These forces are due to both gravity and centrifugal force (the higher the spin speed, the less significant the gravity force is). The CG location of the part may then be separated from the dynamic unbalance of the part using calculations that involve the magnitude of the bearing forces and their phase relationship. Spin balance machines rotate the test item at speeds ranging from 50 RPM to 10,000 RPM and measure the reaction forces acting against the bearings in the machine due to dynamic unbalance (a combination of CG offset and product of inertia).

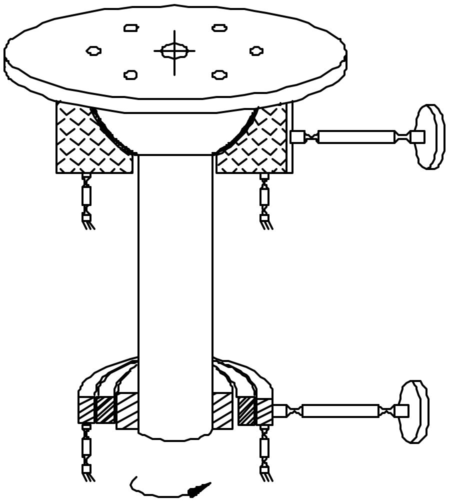


Figure 22 - The spin balance method of CG measurement is expensive and accuracy is often very limited..

At first analysis it might appear that it doesn't make any difference whether CG is measured in a static or a dynamic mode. However, there are a number of considerations which make the spin method of CG measurement unsatisfactory. If you need to measure the longitudinal CG of a long test object, (i.e. 10 meter long rocket), then static measurement is the only way. Spinning such a test object would require tremendous power and generate high winds in the test laboratory. If you are measuring the CG of a partially filled fuel tank, then the fuel will ride up the sides of the tank if you spin it, because of the centrifugal force. This results in an erroneous CG measurement. If the test object has extended solar panels, then centrifugal forces may damage or deflect them. For objects with a large CG offset, the force limits of the transducers will be exceeded if you spin the part. If a part has a very large product of inertia unbalance, CG measurement will be more accurate in the static mode, since the small CG forces do not have to be separated from the larger POI forces. Finally, and perhaps the most important consideration, when an irregularly shaped object is spun, aerodynamic forces cause large variations in the measured CG offset, severely limiting measurement accuracy.

Using a balancing machine to reduce unbalance -- A spin balance machine is the ideal device to balance an object which rotates. For this application, it makes little difference whether the unbalance is caused by CG offset or product of inertia; the goal is to reduce the unbalance to acceptable limits. The balancing machine will instruct the operator as to the ballast weight which must be added to achieve balance. Since the ultimate goal is to reduce unbalance, a 5% error in measurement has little consequence. For large unbalances, this 5% error means that a 20 to 1 reduction is balance is possible for each iteration. It would be hard to do better than that even if the machine were more accurate, since balance weights are not always the correct mass and it is hard to place them in exactly the right location. When balance has been achieved, a 5% error also has little consequence: 5% of a very small unbalance is insignificant.

> Using a balancing machine to measure CG --The situation is totally different when a balancing machine is used to measure CG rather than correct unbalance. In this case, the effect of product of inertia unbalance must be subtracted from total unbalance to obtain CG. Often the product of inertia term is much larger than the CG term, so that the CG is proportional to the difference between two similar large numbers. In effect this is similar to weighing a man by having him drive a dump truck onto a truck scale, obtain the gross weight, and then have the man step out of the truck, and get the net weight of the truck. The difference is the weight of the man. A static CG machine does not respond to product of inertia at all, since there are no centrifugal forces. On a balancing machine, however, CG measurement accuracy will be severely limited if the product of inertia of the object is large. This situation is likely to occur if a tall rocket is measured. The rocket must be mounted vertically in the machine since the windage in the horizontal mode would be so great that the test lab would become a wind tunnel. It is not uncommon is these instances for product of inertia forces to be as much as 100 times greater than the forces due to CG offset. Since these product forces must be subtracted from the CG offset forces, a 5% error in the measurement of the product forces will result in a 500% error in CG measurement!

SUMMARY OF BENEFITS AND SHORTCOMINGS

Benefits

1. By selecting spin speed, machine can be either low sensitivity/high offset range or high sensitivity/low offset range.
2. Sensitivity can be very high at high spin speeds. However, in most cases maximum spin speed is limited by structural limits or the windage of the test object.
3. Measures product of inertia as well as CG.

Shortcomings

1. High product of inertia of test item often obscures CG, limiting accuracy.
2. Air turbulence during spin can produce a large uncertainty in CG measurement Some objects cannot be spun because solar panels or other protrusions would break off due to windage forces.
3. On some test items, spinning alters their CG. Other items cannot tolerate the centrifugal or vibratory force that would occur at the spin speed.
4. Cost of this type of instrument is higher than any other.
5. Fixturing must rigidly support the test item when being subject to relatively high vibratory forces during spin. This makes fixture design much more expensive and difficult than fixtures for static machines.

5.5.2 Moment of Inertia Method of Measuring CG -- The test object is mounted on an inverted torsion pendulum (moment of inertia instrument) and successive moment of inertia measurements made for at least three positions of the test object. The CG location can then be calculated from the small change in MOI which results from moving the principal axis of the object. This method was developed by Space Electronics in the early 1970's and described in SAWE papers #1169 and #1440. As these papers show, this is the least accurate method of CG determination. Its only advantage is the low cost if you already were in possession of a MOI instrument.

Center of gravity is determined on a torsion pendulum by making use of the parallel axis theorem.

If the moment of inertia of the object about axis A-A through its CG is I_A, then the moment of inertia through axis B-B is

$$I_B = I_A + d^2M$$

Where M is the mass of the object and "d" is the distance between axis A-A and axis B-B. Note that the minimum measured moment of inertia of an object occurs when the axis of measurement coincides with the CG of the object.

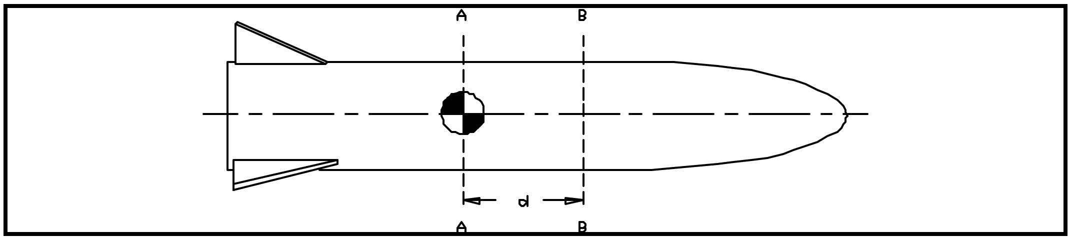


Figure 23 - Center of gravity is determined using the parallel axis theorem.

For the object shown in Figure 23 above the longitudinal CG can be determined by mounting the object in a vee block fixture on a moment of inertia instrument; several moment of inertia measurements are then made at different object positions to determine the object location resulting in the smallest measured moment of inertia.. When the measured moment of inertia is a minimum the CG of the test object is coincident with the axis of measurement.

> This is an extremely tedious procedure, and requires some means of determining part position at minimum moment of inertia. Furthermore, this method only works for single axis CG. A better method is to measure moment of inertia at three known positions and calculate CG from this data using the methods described in the SAWE papers referenced above. Referring to Figure 24, the distance the test part was moved is known, the two values of moment of inertia are measured, the test part is weighed to determine the value of M, and the distance to the CG, d_b , is calculated.
>
> If the displacements between the three measurement positions are made small, then the sensitivity of this method is abysmal. Accuracy of better than 0.1 inch is difficult to obtain. If the displacements are made large relative to the radius of gyration of the test object, then the accuracy improves from a theoretical standpoint. However, torsion pendulums do not operate successfully with large CG offsets, due to the gravity pendulum error, so that the increased measurement error partially offsets the gain in sensitivity, and the accuracy of measurement is still worse than other methods.

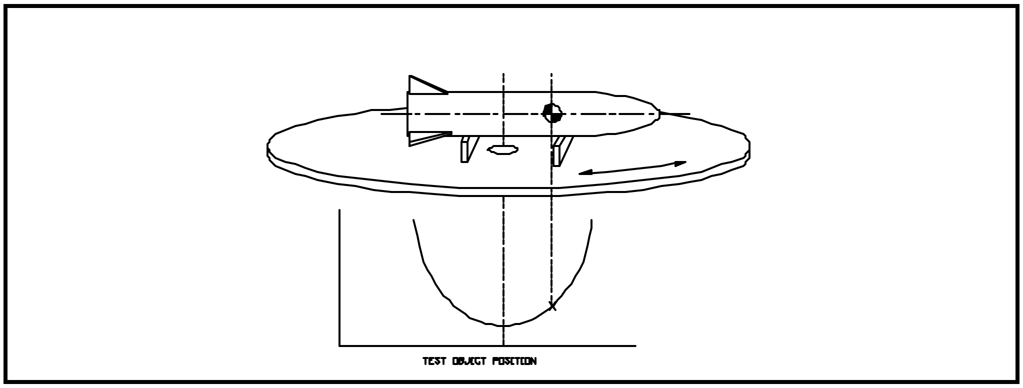


Figure 24 - Test object is repositioned in vee blocks until point of minimum moment of inertia is determined

SUMMARY OF BENEFITS AND SHORTCOMINGS

Benefits

1. If you already have a moment of inertia instrument, it can be configured to measure CG with relatively little extra cost.
2. Machine performs with creditable accuracy when measuring small diameter solid steel test weights (unfortunately very few real test objects are of this type).

Shortcomings

1. Accuracy is poor for realistic test parts (ones whose radius of gyration is considerable larger than the CG offset from the center of the instrument).
2. Each measurement requires the test object to be positioned in three different locations, so that an elaborate and expensive fixture is required.
3. The instrument does not give a direct readout of CG, so that corrections cannot be made to the object and the resulting CG shift observed.
4. Error increases dramatically if the machine is even slightly out of level.
5. Test items containing fluids cannot be measured accurately
6. The test is tedious to run.