6.0 MOI measurement

Every engineer knows he can measure moment of inertia by hanging an object from a wire, twisting it to start it oscillating, and then timing the period of oscillation. However, anyone who has actually tried this finds that the object swings from side to side and rocks up and down rather than rotating smoothly about an axis, making it difficult to accurately time the period of oscillation. Furthermore, there are a number of practical problems involved in hanging most test articles from a wire. How do you attach the wire to the object? Where do you attach the upper end of the wire (particularly if the object weighs more than 1000 kg)? How do you calibrate the device, and what do you do to correct for the change in calibration when the weight of the test object stretches the wire?

6.1 Inverted Torsion Pendulum Modern moment of inertia instruments consist of an inverted torsion pendulum which oscillates in a rotational sense and a means of measuring the exact period of oscillation of the torsion pendulum. Instead of hanging from a torsion rod or wire, the test object rests on a precision rotary table attached to the top of the instrument. Low friction bearings support the table and payload while constraining the motion of this torsion member to pure rotation. Air bearings provide the best performance. Unlike custom-made hanging wire, trifilar, or compound pendulum systems, measurements are made about a well defined axis, a minimum amount of fixturing is required, and elaborate computational techniques are not necessary.

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The measurement of the moment of inertia of the test part is based on the change in the natural frequency of oscillation of the torsion pendulum resulting from the addition of the test part mass. This change in natural frequency is compared with the change in natural frequency which occurs when a calibration mass of known moment of inertia is placed on the instrument.

<u>Step 1</u> The object is secured to the table with its CG aligned with the axis of the bearing. The part is rotated and released. It will then oscillate about the fixed axis of the instrument and the total time for one complete oscillation can be displayed on a digital period counter. The total combined moment of inertia of the test object, its fixture, and the instrument itself can be calculated from the formula:

$$I_x = CT_x^2 \qquad \text{TOTAL MOMENT OF INERTIA}$$

where I_x is equal to the total moment of inertia, C is the calibration constant of the instrument, (a function of its torsional stiffness), and T_x is the period of oscillation in seconds.

<u>Step 2</u> The test object is then removed from the instrument and the "tare" moment of inertia of the instrument and the fixture determined by measuring the oscillation time period without the test object.

$$I_o = CT_o^2 \qquad \text{TARE MOMENT OF INERTIA}$$

Step 3 The moment of inertia of the test object is then the difference between the total inertia and tare inertia.

$$I = I_x - I_o \qquad \text{NET MOMENT OF INERTIA OF OBJECT}$$

in order to establish the value of the calibration constant, C, of the instrument, MOI calibration standards are measured. MOI calibration standards are precision weights of simple geometry, known mass and known physical dimensions.

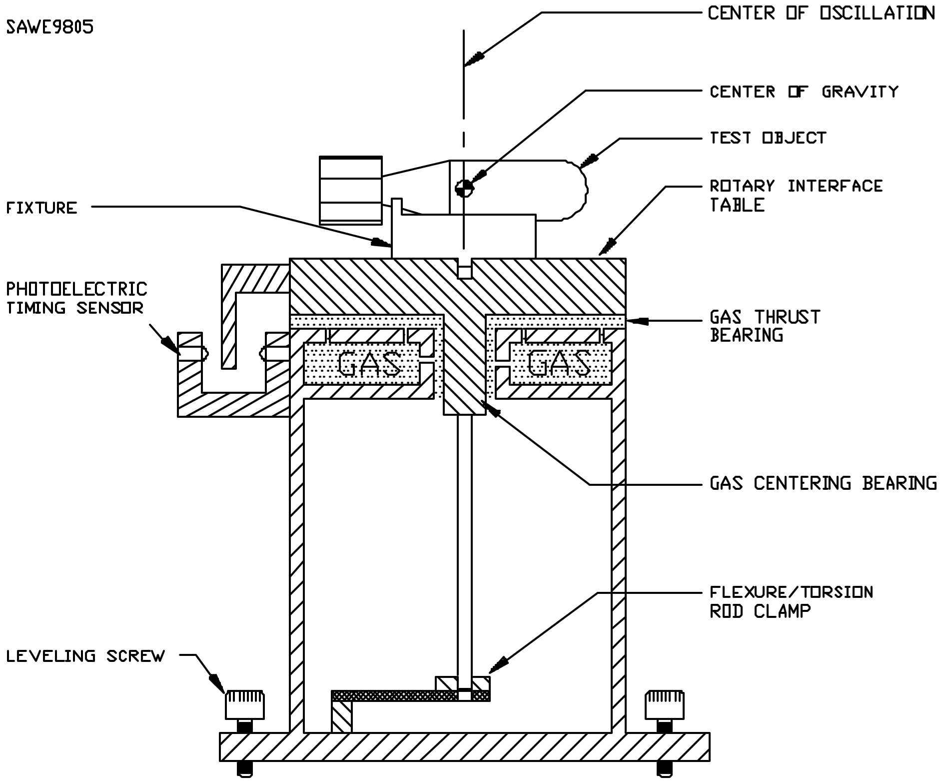


Figure 25

The calibration procedure is identical with the procedure for measuring the moment of inertia of a object of unknown MOI, except that in the computation, the inertia is a known quantity and the value of the calibration constant is the unknown which must be solved for.

Because the weight of the object is supported by the air bearing, these instruments are linear over a wide range of test part weight and moment of inertia. Only a single calibration measurement is required to establish the value of the calibration constant used for all measurements.

6.2 Description of MOI Instrument

A greatly simplified cross section view of a MOI instrument is shown in figure 25. The instrument consists of three basic parts: an air bearing, a torsion rod, and a photocell system for timing the period of oscillation. The test part is first mounted in a test fixture (designed and fabricated by the user) which locates the test part in proper orientation to the measurement axis of the instrument and couples the test part rigidly to the interface table of the instrument during the testing. The test fixture itself is aligned with the interface table of the instrument through the use of a precision pilot in the interface table.

To measure the moment of inertia of the test part, gas pressure is applied to the storage cavities in the gas bearing. The gas then flows through small diameter orifices to the gap between the gas bearing and the interface table, floating the interface table on a film of gas. The damping in the torsion pendulum therefore consists only of the internal damping in the torsion rod. The interface table is temporarily twisted in a counterclockwise direction until it contacts a stop; then it is sharply released, resulting in an oscillatory motion of the interface table due to the stiffness of the torsion rod (which is rigidly attached to the interface table and clamped to a flexure at its lower end). The time period of oscillation is then measured by connecting a digital period counter to the photoelectric timing sensor. As the interface table oscillates, a timing pulse is emitted as the table reaches its mid-point of oscillation when traveling in a clockwise direction. The first timing pulse starts the counter, a second timing pulse stops the counter exactly one period later.

6.3 Keeping the Test Part CG Coincident with the Rotational Axis It is important that the center of gravity of the test part and fixture be positioned so that it is as close as possible to the rotational axis of the torsion pendulum. Otherwise measurement error will increase for a number of reasons.

Most significant of these is the so-called "gravity pendulum" error which occurs because the axis of rotation of the torsion pendulum can never be made exactly vertical. If a part is rotated about an axis which does not fall on its center of gravity, then the force of gravity acting through the center of gravity will tend to bring the center of gravity to its low point (i.e. the direction in which the axis of rotation of the torsion pendulum is tilted), resulting in a change in the effective calibration constant of the instrument. This effect can be minimized by leveling the table and re-positioning the test part so that its center of gravity lies close to the rotational axis of the instrument.

A second source of error with offset center of gravity is the classical axis translation error. Mathematically, this increase is equal to the test part mass times the square of the offset distance. Since this increase can be exactly calculated, it can be subtracted from the measured moment of inertia; therefore it does not constitute an uncertainty. For small offsets, (less than .01 times the radius of gyration) this effect is negligible. For larger offsets, instruments are available which measure both CG and MOI. The software then automatically reports the MOI about the center of oscillation and about the CG. To utilize this feature, the test part must be weighed.

If it is desired to measure a test part through a point other than its center of gravity, the most accurate method of accomplishing this is to first measure the moment of inertia of the part about its center of gravity and then mathematically translate this measured value to the new axis by adding the translation factor R^2 M where R is the distance between the CG axis and the desired axis, and M is the mass of the test part. This procedure also requires the weighing of the test part.

6.4 Effect of air mass - For large lightweight objects, the measured mass properties are often different from the calculated values. In particular, measured moment of inertia can be 10% to 20% larger than calculated. The reason for this is that air has significant mass and alters the mass properties in two ways:

1. Air trapped **inside** the payload will increase its mass by an amount equal to the unoccupied volume in the payload times the density of air (0.0754 pounds per cubic foot). For example, the air trapped in a 4 foot diameter, 2 foot long satellite weighs approximately 4 lbs. We call this the <u>entrapped</u> air effect.

2. Air dragged or pushed along by any protrusions on the outer surface of the payload can dramatically increase moment of inertia. For example, the roll moment of inertia of a missile flying in air is much larger than the roll MOI of the missile in a vacuum. We call this the <u>entrained</u> air effect.

How you handle this difference depends on whether the payload operates in the vacuum of space or in air. If the payload flies in a vacuum, then measured values must be decreased to eliminate the effect of air mass. The best way of doing this is to make a second measurement in helium and then extrapolate the value in vacuum (see SAWE paper No. 2024 by Boynton and Wiener). Calculated values remain unchanged.

If the payload flies in air, then measured values remain unchanged and represent the true mass properties.

6.5 Minimum MOI which can be measured

Moment of inertia instruments have an amazing dynamic range. A MOI instrument which is designed to measure objects weighing up to 3000 pounds can often detect the change in MOI due to the addition of an object weighing 0.1 pound. However, accuracy is reduced when the MOI of an object is smaller than the tare MOI of the instrument. The error is primarily due to thermal expansion and contraction of the instrument and fixture during the time between tare and object MOI measurement. For example, if a payload has a MOI of 10 lb-in^2, and the instrument has a tare MOI of 10,000 lb-in^2, then a 0.1% change in tare due to an ambient temperature change will result in a 100% error in the measured MOI of the payload. Reducing short-term temperature change can increase the usable range of an instrument. We have found that improved accuracy can be achieved by simply shutting off the heating or air conditioning system during the interval of time between tare and object measurement. Temperature control systems that frequently cycle are not desirable.

6.6 Damping

Air bearing MOI instruments themselves have very small losses, and the effect of this damping can generally be ignored. For payloads which introduce significant damping though air turbulence while oscillating, the actual period of oscillation is greater than the undamped natural period by an amount determined by the damping ratio, z. If the torsion pendulum is being used as an instrument to measure moment of inertia, then the measured moment of inertia will be greater than the true value. This error can be eliminated if the following equation is used in place of equation XX. The quantity z^2 is the error.

$$I = C T^2 (1-z^2)$$

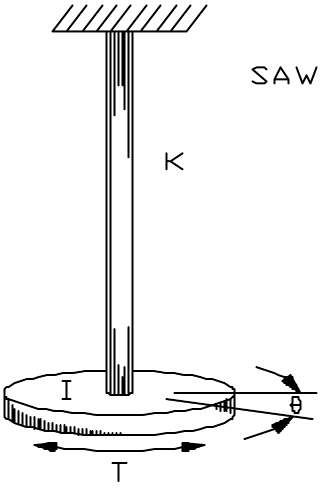


Figure 26

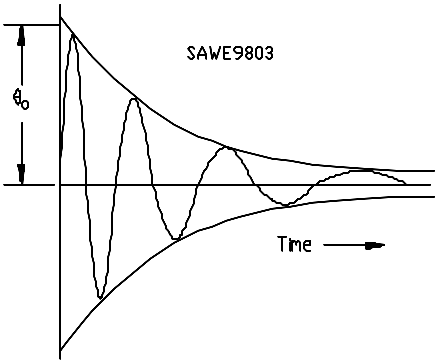


Figure 27

In order to make use of this equation, the value of the damping ratio, z, must be determined. This is accomplished by noting the rate at which the amplitude of oscillation decays. If we define the logarithmic decrement as the natural logarithm of the ratio of any two successive amplitudes, then the log decrement, d, of the starting amplitude, a_o, as compared to the peak amplitude, a_n, after n cycles have elapsed is given by the equation:

$$d = 1/n (\ln \ a_o/a_n)$$

For small values of z, the logarithmic decrement, d, can be related to z by the following relationship.

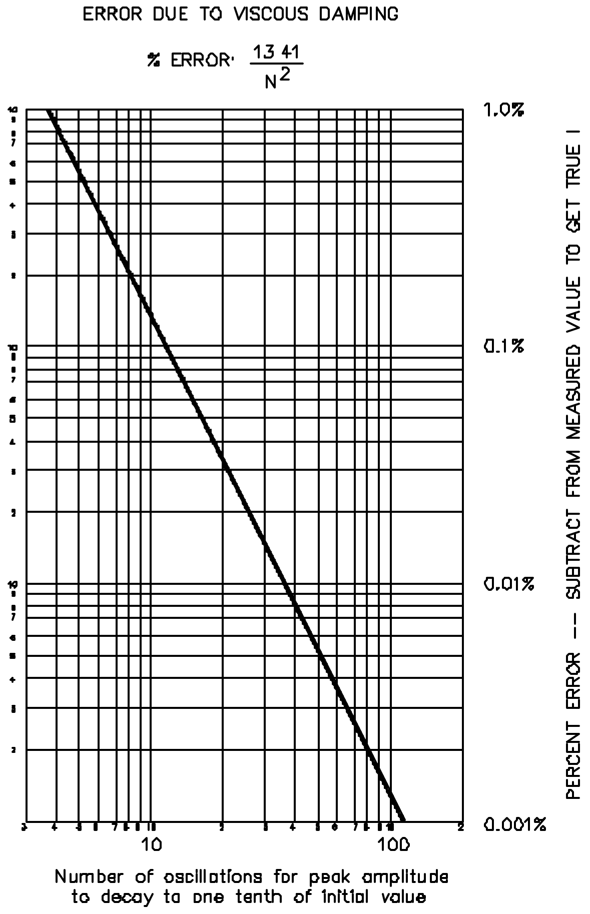
$$d = 2 B z$$

If we now count the number of oscillations of our torsion pendulum, n, for a decay in peak amplitude of 10/1, we may combine the above equations and solve for the error resulting from damping.

% error due to damping : 100 z^2

$$\% e = \frac{(\ln 10)^2 100}{(2\pi n)^2} = \frac{13.41}{n^2}$$

A graphical solution to this equation is given in figure 28. To correct the measured value of moment of inertia, the amount shown on the graph should be subtracted from the measured value to yield the true value. Note that the error is insignificant if more than 50 oscillations are required for the amplitude to decay to one tenth of its original value.



6.7 Hanging wire torsion pendulum Although hanging wire pendulums are not accurate enough for satellite and missile measurements, they are useful for measuring the MOI of an aircraft. In fact, no torsion pendulum instrument currently exists which is large enough for aircraft measurement, so there is no choice. Fortunately, the MOI tolerance of an aircraft is not critical and the resulting accuracy is acceptable.

This method consists of hanging an object from a wire, twisting it to start it oscillating, and then timing the period of oscillation. Although it sound like a simple device, the structure required to support the upper end of the wire can be very expensive, and some accurate means is required to time the period of oscillation. One problem with the hanging wire method is that the object swings from side to side and rocks up and down rather than rotating smoothly about an axis, making it difficult to acquire accurate time period data.

It is essential that the center of gravity of the object be aligned horizontally with the center of the torsion rod. Otherwise, the moment the object is released, there will be a couple generated and the motion of the pendulum will be sideways as well as torsional. There is a serious practical problem when measuring heavy objects using this method: how do you attach the object and adjust its position so the CG is in the center of the rod?

A single hanging (steel) wire has a torsional stiffness (k) which is proportional to the fourth power of the diameter:

$$k = \frac{1,178,000 d^4}{L} \; inch-lb/radian$$ where L = length and d = diameter (inches)

The equation of motion for this pendulum is:

$$I = Ct^2 = \frac{Kt}{4\pi^2}$$

However, the actual period of oscillation will not follow this formula because the wire stretches under load, and there will be both a swinging pendulum effect and a rocking pendulum. Typical measurement uncertainty with this method is about 3%.

It is possible to hold the object level and eliminate the rocking mode of oscillation by using three wires rather than one. This is called a trifilar pendulum. It is the method most commonly used for large heavy payloads such as aircraft. Unfortunately, using three wires introduces a nonlinearity. As the pendulum twists, the wires restrain each other, causing the object to lift slightly. So the torsion constant becomes a function of test object weight as well as torsional stiffness. The oscillation period will increase as oscillation amplitude decreases. Therefore, it is necessary to use a timing system that triggers at a specific amplitude.

When using a single wire system, it is obvious when the object CG is misaligned, because the object will hang at an angle. However, if you use a trifilar pendulum, the object will not tilt appreciably, so some independent means is necessary to align the object CG such as incorporating load cells into the wires to adjust for equal loading of all three wires.

Shown here is a Space Electronics POI series Spin Balance Machine with a 12,000 pound capacity. An "L" Fixture permits 3-axis measurement by rotating the test object about its horizontal axis. The unique gas bearing design of the Space Electronics POI series Spin Balance Machines allows measurement of dynamic unbalance, product of inertia (POI), moment of inertia (MOI), and center of gravity (CG) with a single setup. Its slow spin speed (10 - 300 RPM) minimizes centrifugal forces on the payload during dynamic unbalance measurements.

7.0 Measuring Product of Inertia

There are two methods which can be used to measure POI:

1. The object can be rotated in a spin balancing machine, and the reaction forces measured against the bearings. POI can then be determined by performing calculations that involve the magnitude of the bearing forces and their phase relationship. For the measurement of rockets and satellites, the spin speed is usually about 100 RPM. This minimizes the effect of air turbulence. A special high sensitivity spin balance machine is required, which differs greatly in construction from the type of high speed balancing machine that is used to measure automobile crankshafts and jet engine rotors.

2. Objects such as control fins and satellites with extended solar panels cannot be measured using the spin method, because of the large, non-repeatable errors which are introduced by the entrained and entrapped air and turbulence. In these instances, product of inertia can be determined by making a series of moment of inertia measurements with the object oriented in 6 different positions. Product of inertia can then be calculated using formulas which involve the rotation angles of the different fixture positions. Moment of inertia is measured by oscillating the object on a torsion pendulum. Since the object moves very slowly during this measurement, there are negligible centrifugal and windage forces exerted on the object. Furthermore, the mass of the entrapped and entrained air can be compensated for by making a second set of measurements in helium, and extrapolating the data to predict the mass properties in a vacuum

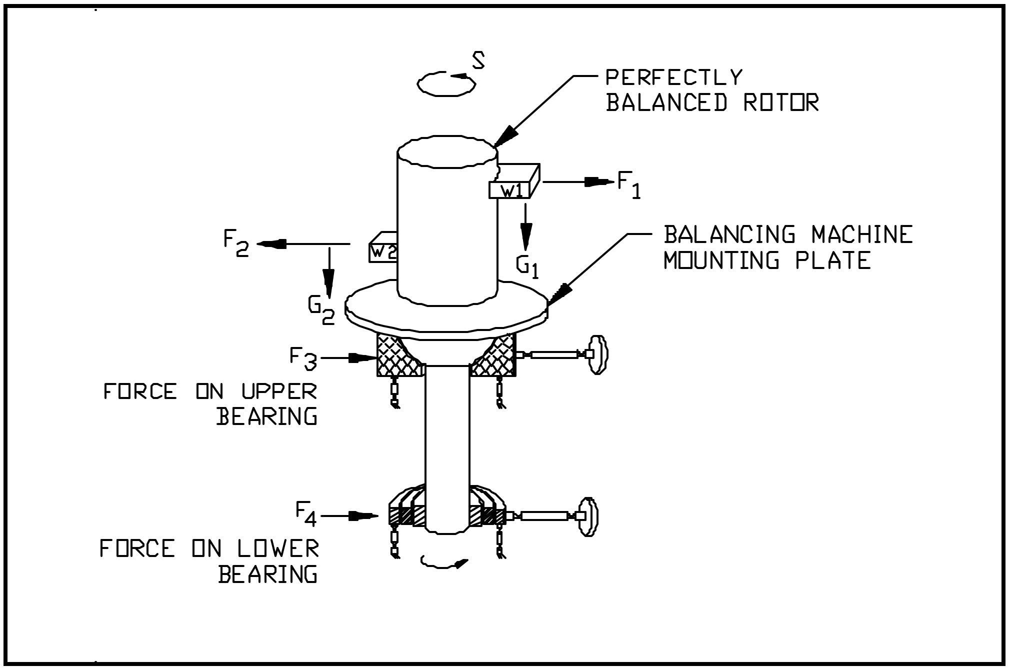
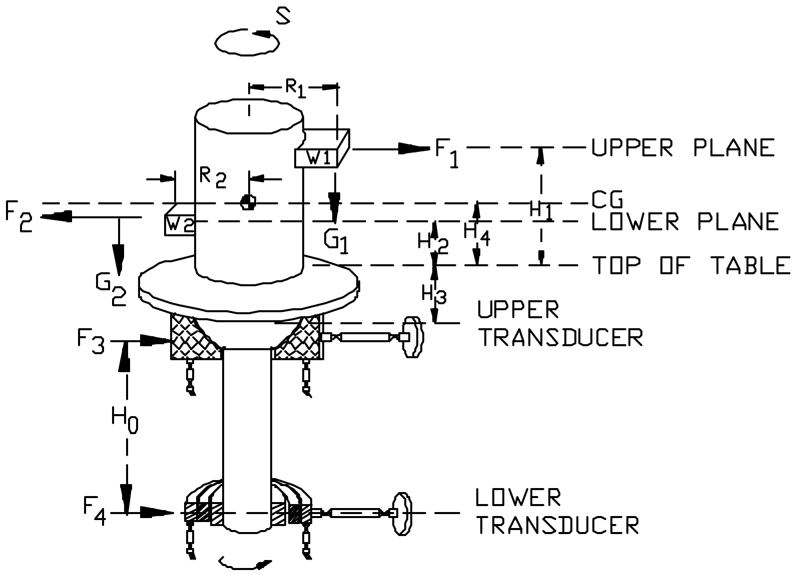


Figure 29

.SPIN BALANCE METHOD

Product of inertia is generally measured using a spin balance machine. In this type of machine, the object is rotated at a fixed speed, and the reaction forces against the upper and lower spindle bearings are measured. Product of inertia is then calculated automatically by the machine's on line computer, using formulas that involve the vertical spacing between the upper and lower bearings, and the height of the CG object above the mounting surface of the machine. The CG location of the part may be separated mathematically from the dynamic unbalance of the part.



7.1 Balancing Machine Theory -- When the test object spins, there are two forces acting through the CG of the object: gravity forces acting downward and centrifugal forces acting horizontally (the higher the spin speed, the less significant the gravity force is). The magnitude of the downward gravity force is equal to the weight of the object in pounds (M1). The magnitude of the horizontal centrifugal force is:

Centrifugal force (lbs) $F_1 = M_1 \times R_1 \times S^2$

where M_1 = mass of unbalance in slugs
R_1 = radius of CG in feet
S = speed in radians per second

Converting the mass into weight and the speed into RPM:

$$(\text{lbs})\ F_1 = \frac{W_1 \text{ x } R_1 \text{ x } (\text{RPM})^2}{35207}$$

where W_1 = weight of unbalance mass in lbs
R_1 = radius of CG of unbalance in inches
RPM = speed of rotation in RPM
35207 = constant to transform units

The forces applied to the bearings of the balancing machine will depend on the geometry of the balancing machine and the type of unbalance. If neither a CG offset or a product of inertia unbalance is present in the rotating test object, then the forces on the bearings will be zero. A product of inertia (with no CG offset) results in equal forces being applied 180 degrees out of phase on the two bearings (the couple due to the product of inertia is offset by an equal couple on the bearings of the balancing machine). A CG offset will cause a different force to be applied to each bearing. In order to evaluate the accuracy of a balancing machine in measuring CG, it is necessary to know the relative values of POI and CG offset. For a flywheel, POI is usually small, and the balancing machine will be able to accurately measure CG offset. For a tall rocket, the reverse is true.

If the goal of the measurement is to ballast the test object for minimum POI and CG offset about a particular axis, then the measurement becomes much more accurate. As the unbalance is reduced in successive iterations, the residual unbalance can be measured on a more sensitive scale and the magnitude of POI can be reduced so that some CG sensitivity results.

To calculate the CG offset from the measured bearing forces, we can make use of the fact that the sum of the moments around each transducer must be zero (since the system is stable). Independent calculations can be made at the X and Y axes, and the resultant of these two calculations determined to yield the magnitude and angle of the CG offset.

7.1.1 Two transducers are required to separate POI from CG offset In order to separate the unbalance due to POI from the unbalance due to CG offset, it is necessary to use a force transducer on each of the spindle bearings in the balancing machine. Then both CG offset and product of inertia are obtained in a single measurement. If two measurements are made, then it is theoretically possible to use a single transducer to measure both CG and product of inertia. Static measurements are first made to determine CG offset. Since the gravity component of CG offset does not change with speed, it could be concluded that this component can be separated from POI. In the real world, however, it doesn't work. The reason is that the large forces resulting from the CG offset must be subtracted from the smaller forces due to POI. Since the CG offset is measured under static conditions where forces are small, there is an error. This small static force error becomes a very large error when spinning at 100 RPM.

7.1.2 Errors due to air turbulence The force on the transducers of the balancing machine increases as the square of spin speed. In some POI machine designs, the transducers chosen are velocity pickups whose sensitivity also increases with speed. The net effect in this case is that the sensitivity of these balancing machines increases as the cube of the speed. At first it might seem that there would no limit to the sensitivity which could be obtained with this method. However, there are a number of factors which limit the speed of the machine.

The machine must be rigid enough not to go into resonance at high spin speeds. The test item must be strong enough to withstand the centrifugal forces. The outer surface of the test item must be smooth and round, so that the test item will not fan the air and create turbulence. Solid metal shafts, such as turbine rotors, are smooth and round and may be spun at a high speed when balancing them. Aerospace hardware, however, usually cannot be spun above 200 RPM without damage occurring, and windage forces on the irregular outer surface of the vehicle increase as the square of the spin speed, so that the random vibration produced by air turbulence provides another limitation. This vibration limits the real sensitivity of the balancing machine.

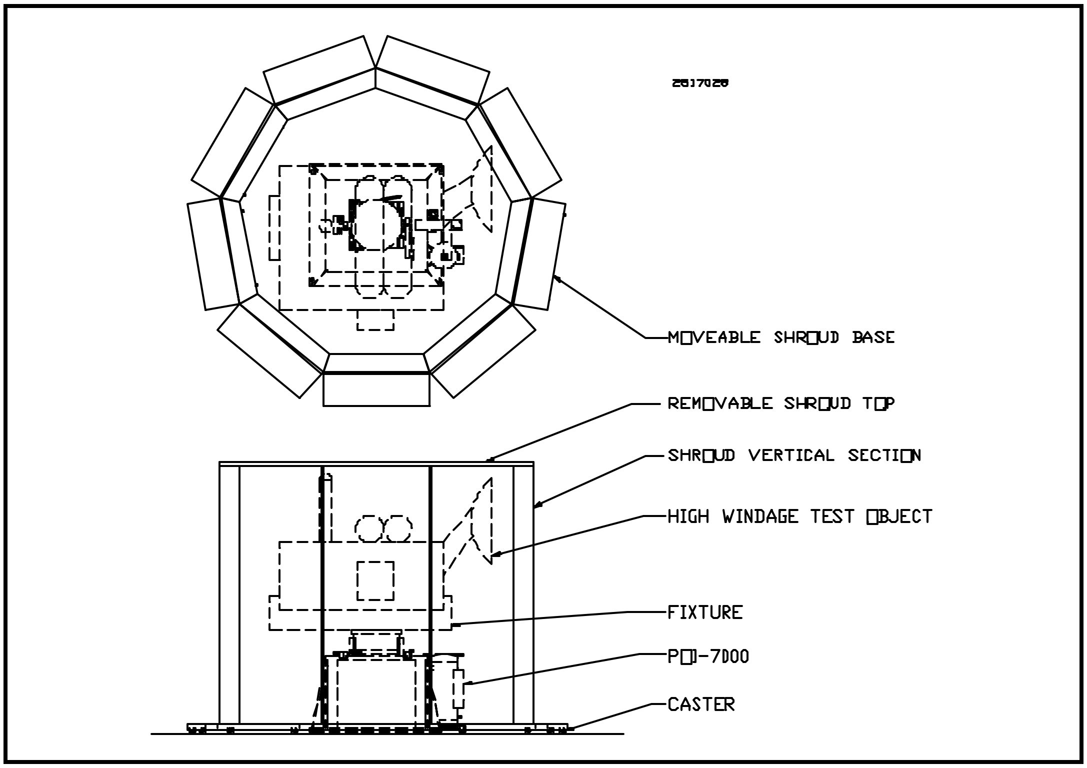


Figure 31 Typical Shroud Installation

The error due to air mass can be minimized by surrounding the spin balance machine with a shroud. Typical shrouds are cylindrical in overall shape and adjustable in height and radius so that they can be made to enclose the spinning object as closely as possible. Additional improvement can be obtained by placing a circular cap on the shroud to close the overhead opening.

The objective is to enclose the smallest possible volume of air, while allowing the test object to spin with adequate clearance. The walls of the shroud must approximate a smooth cylinder as much as possible. This will allow all of the air within the shroud to turn at the same rate as the test object and minimize the resulting forces on the transducer. Note: the need for a shroud is not a special requirement of a particular balancing machine. Any balancing machine manufactured by any company will have the same limitation.

SUMMARY OF BENEFITS AND SHORTCOMINGS

Benefits

1. By selecting spin speed, machine can be either low sensitivity/high offset range or high sensitivity/low offset range.
2. Sensitivity can be very high at high spin speeds. However, in many cases maximum spin speed is limited by structural limits or the windage of the test object.
3. Measures CG and well as product of inertia.

Shortcomings

1. Air turbulence during spin produces uncertainty in measurement;
2. Some objects cannot be spun because solar panels or other protrusions would break off due to windage forces, or they cannot tolerate centrifugal or vibratory force that occurs at the spin speed.
3. Spin balance machines are expensive.

7.2 Moment of Inertia Method of Measuring POI

This method uses a torsion pendulum to determine POI by making use of the relationship between POI and MOI of an object. Special fixtures must be constructed to move the object to a number of positions while keeping both the object and the fixture CG near the center of oscillation. The moment of inertia of the object is measured in each orientation. The tare MOI of the fixtures must then be measured and subtracted from the measurement with the object. The net MOI of the object in the different orientations is then used to determine the POI of the object The calculations are quite complex, so an on-line computer is used.

To better understand the concept, refer to Mohr's Circle on the following page. The axes of minimum and maximum moment of inertia correspond to the axes where the product of inertia is zero. These are called the principal axes. The product of inertia is a maximum at an angle of 45^{o} from these principal axes.

If the test part were fixtured so that it could be rotated through an angle C about a horizontal axis (i.e. the Z axis) and MOI measured about numerous axes in the X-Y plane, including the X and Y axes, the MOI would be found to vary sinusoidally. If the angle C ranges over 180 degrees, the maximum and minimum values of MOI can be seen in a plot of MOI vs C.

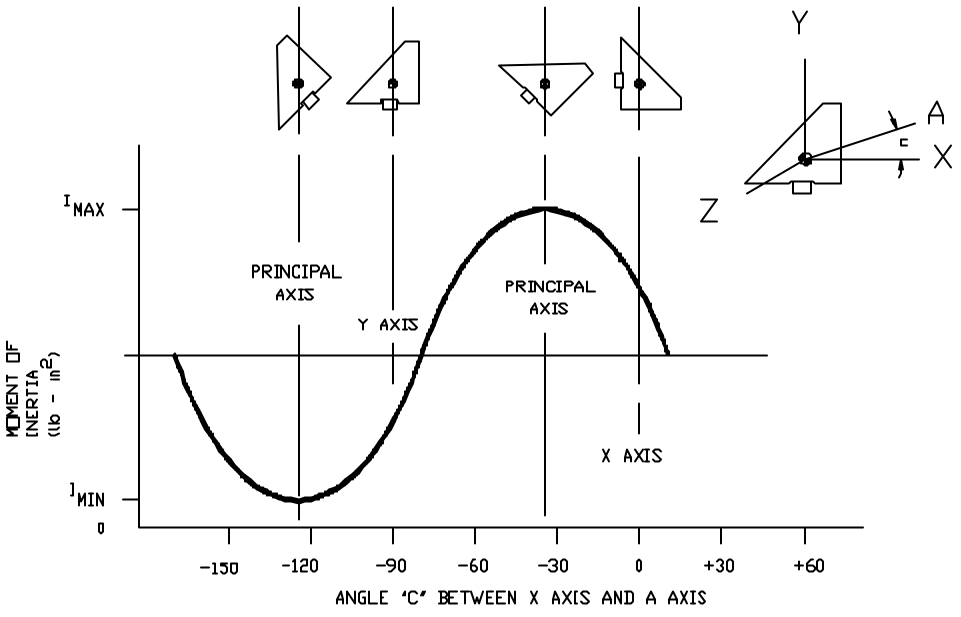
The axes about which the maximum and minimum MOIs are measured are the Principal Axes. For all other axes the moment of inertia I_{Axy}, about an axis (A) in the X-Y plane at an angle C from the +X axis, and the product of inertia P_{xy}, are related through the equation:

$$I_{Axy} = I_{yy}\sin^2 C + I_{xx}\cos^2 C - P_{xy}\sin 2C$$

Solving this equation for P_{xy} forms the basis for the MOI method of POI determination.

$$P_{xy} = \frac{(I_{yy}\sin^2 C + I_{xx}\cos^2 C - I_{Axy})}{\sin 2C}$$

7.2.1 NUMBER OF MOI MEASUREMENTS For the general case, the total number of MOI measurements needed for POI calculations is nine: three in each of three mutually perpendicular planes. If the intersections of these planes are selected to be the coordinate axes, then the MOI about each of these axes will be common to two planes, thus reducing the total number of measurements to six: three about the X, Y, and Z axes, and three about axes at 45 degrees between the X-Y, Y-Z, and Z-X axes. If vacuum data is required, the same six MOI measurements must also be repeated in a helium atmosphere.



7.2.2 Mohr's Circle for Moments of Inertia

Given:

(1) The moment of inertia values $I_{X'}$ I_Y for an object about its center of gravity, where the center of gravity lies at the origin of a set of mutually perpendicular axes X-Y.

(2) The corresponding value for the product of inertia, $P_{XY'}$

Mohr's circle is then constructed using the layout geometry shown below. The following information may then be obtained.

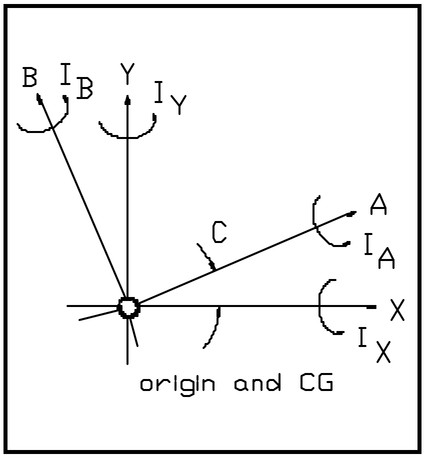


Figure 33

(1) The location of the principal axes about which the moments of inertia are maximum and minimum and the products of inertia are zero.

(2) The corresponding maximum and minimum values of moments of inertia.

(3) The moments and products of inertia for any other set of mutually perpendicular axes A-B whose origin lies at the center of gravity of the given object and rotated C degrees from the original axes X-Y. Reference, the figure to the right.

(4) The maximum values for the products of inertia about axes located 45° from the principal axes.

Layout Geometry

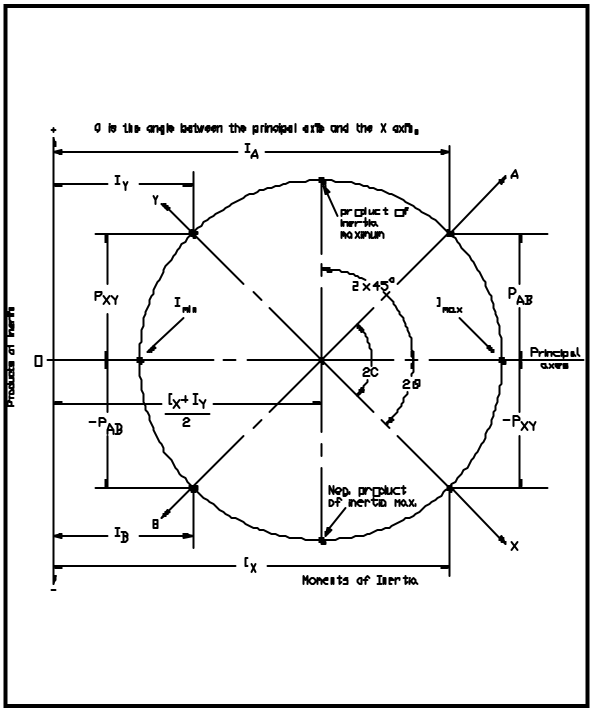


Figure 34

The radius of the Circle is:

$$R = \sqrt{\left(\frac{I_X - I_Y}{2}\right)^2 + (P_{XY})^2}$$

SUMMARY OF BENEFITS AND SHORTCOMINGS

Benefits

1. The cost of a moment of inertia instrument is much less than the cost of a spin balance machine. If you already have a moment of inertia instrument, it can be configured to measure POI at a relatively minor cost. (However, the cost of fixturing and labor is considerably greater than for spin balancing machine measurements).
2. This method puts a minimum stress on the object being measured and is ideal for fragile satellites which cannot be spun on a balancing machine.
3. The effect of air mass and turbulence is minimized.

Shortcomings

1. Accuracy is poorer than with a spin balance machine.
2. Each measurement requires the test object to be positioned in up to nine different positions, so that elaborate and expensive fixturing is required.
3. The test is very tedious to run and labor cost is high. Measuring the POI of a single object can require as much as 10 hours including set up.
4. The instrument does not give a direct readout of POI, so that corrections cannot be made to the object and the resulting POI shift observed.
5. Error increases dramatically if the machine is even slightly out of level.

8.0 Considerations in Choosing a Mass Properties Instrument

There are a wide variety of mass properties measuring instruments available today. The choice of which one to use in part depends on what properties you want to measure, the accuracy required, the degree of automation required, and budgetary restrictions. If you are measuring CG, you need to determine whether you need to measure along a single axis, or along more than one axis (some CG instruments are only capable a single axis measurement). In addition, you need to choose the size of the instrument. This is usually governed by the weight of the largest object you need to measure.

Since CG sensitivity decreases as the size of the instrument gets larger, the selection of an instrument may involve a tradeoff. Frequently, management attempts to purchase an instrument which will serve present and anticipated needs. This may result in the selection of too large an instrument, resulting in limited accuracy for the present requirements.

8.1 What properties you want to measure -- There are instruments available which measure only one mass property; Center of Gravity Location (CG), Moment of Inertia (MOI), Product of Inertia (POI), Weight or Mass. Many instruments can measure 2 or more of these properties due to inherent characteristics or by adding relatively low cost options. Using a combined function instrument often eliminates the effort, cost, and risk involved in moving the test item to another instrument and/or fixture. Some of the most common dedicated and combined instruments measure:

Dedicated

CG (measured statically) only*
MOI only
POI and CG (measured dynamically)
Weight only

Combined

CG and Weight
MOI and CG (measured statically)
POI, and CG (measured both statically and dynamically)
MOI, POI, and CG (measured dynamically)
MOI, POI, and CG (measured both statically and dynamically)

* Static measurement of CG depends on measuring a moment balance condition where forces are generated only by gravity. Dynamic measurement of CG involves moments generated by gravity *and* by centrifugal forces generated as the test article rotates.

<u>Need to measure both CG and MOI</u> -- In order to measure MOI, you will need an instrument with a gas bearing rotary table. Instruments such as the Space Electronics KSR series have this combined capability.

<u>Need to measure both CG and weight</u> -- Multiple point weighing type instruments have this capability. These instruments are available with three load cell technology or with the more accurate force cell technology (Space Electronics WCG series).

<u>Need to measure both CG and POI</u> -- If you have a requirement to measure dynamic balance as well as CG, then a spin balance machine would be a good solution, particularly if your goal was to ballast the test object for minimum POI and CG offset about a particular axis. Space Electronics POI series instrument are available with a separate static CG feature. This improves CG measurement accuracy over what would normally be available with a spin balance machine.

<u>Need maximum CG accuracy</u> --Instruments with a gas bearing rotary table and force restoration moment measuring technology are the most accurate. Space Electronics KSR series instruments are of this type.

<u>Need an explosion proof design</u> -- All types are available with this option. Since repositioning CG instruments are purely mechanical, they are intrinsically safe in explosive environments.

<u>Need to measure CG of objects weighing more than 25,000 pounds</u> -- Three-point reaction force machines have been made to measure objects as heavy as the space shuttle. Gas bearing rotary table machines are limited to about 25,000 pounds because of practical problems in building gas bearings larger than this.

9.0 Weight Measurement

9.1 Types of scales Purely mechanical scales have largely gone the way of the dinosaur. Nearly all scales currently in use have electronic digital displays and computer interface capability. The most common scales still use strain gage load cells with typical accuracy of one part in 2000. These scales lend themselves to computer interfacing at relatively low cost. Pulsed DC power supplies and linearization circuits have allowed accuracies to one part in 5000 at slightly higher cost.

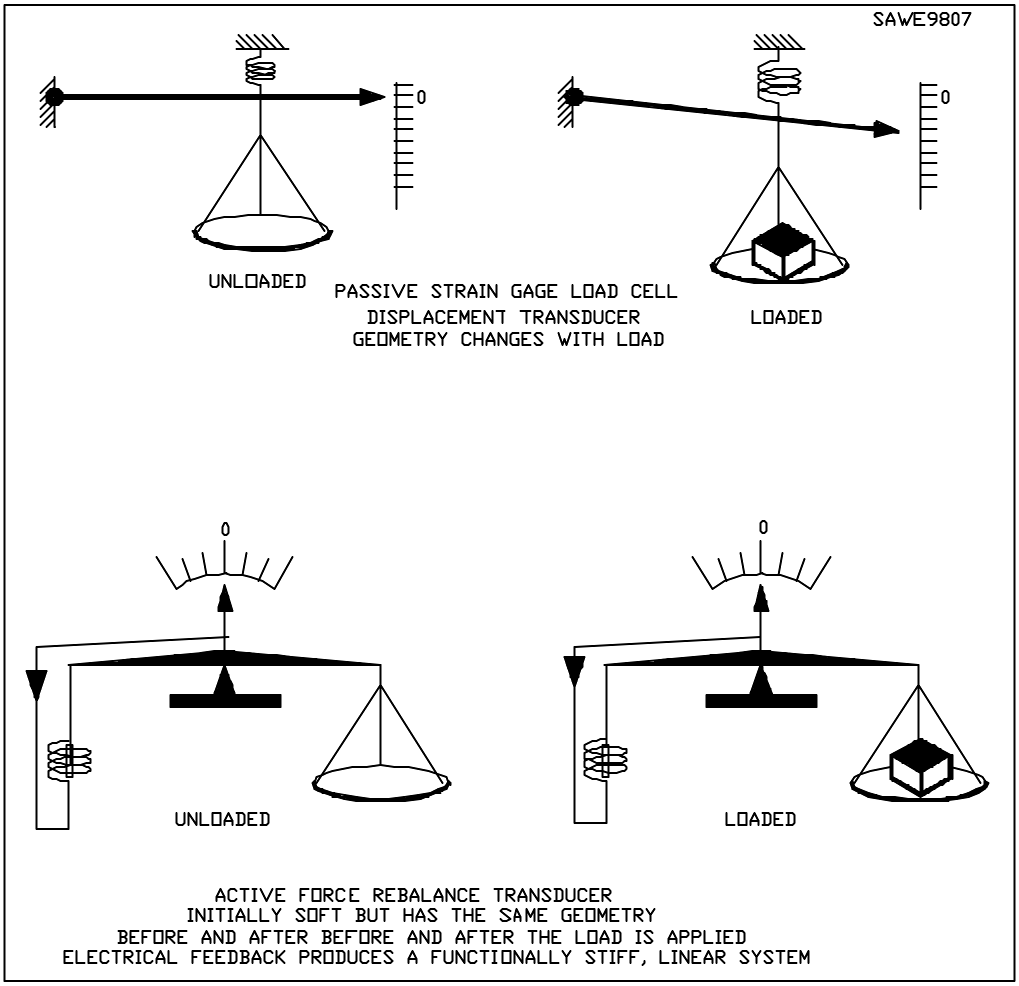
The next level of price with improved rangeability comes with new ceramic capacitive strain gages (for purposes of this paper, we will define rangeability as the ratio between load capacity and accuracy). These scales can be made with accuracies up to one part in 50,000. Transducer stiffness is comparable with strain gage beam cells. These transducers have one drawback: they can be damaged if a hard object is dropped on the scale, since the ceramic spring is brittle and cannot withstand shock. Some newer scales using this technology incorporate spring shock dampers to eliminate this problem.

The biggest innovation has been the application of force restoration technology to weight measurement. This technology has been in use since the late 1950's in both electronic and pneumatic process control transducers. The newest generation of electronic force rebalance transducers can achieve accuracies on the order of one part in 10 million in laboratory balances. In the more common bench scale ranges up to 25 lb, the accuracy can approach one part in 1 million. In the larger sizes, 75 to 13,000 lb, accuracies are typically one part in 25,000 to one part in 50,000. At this time, the maximum load ratings available are on the order of 13,000 lb.

These scales are highly programmable to accommodate many weighing conditions: stability (i.e. animal weighing), parts counting on weight basis, selectable units of measurement, etc. They are fully compatible with computer interfacing. The major disadvantages are price, and slow response time. The slow response time is due to the fact that a closed loop rebalance circuit is used.

9.2 Force Restoration Principle When a load is applied, the transducer deflects. A current driven restoring force is applied through a closed loop control system until the unloaded geometry is restored. The applied current is then related to the applied force. Since the loaded geometry after the restoring force is applied is the same as the unloaded geometry, the transducer is inherently linear like the time honored balance beam scale. This is unlike the strain gage load cell which relies on the deformation of the sensitive spring element to generate an output. High accuracy mass properties measuring instruments for static CG and moment measurement use this force restoration technology.

9.3 Comparison of scale types The table below compares scales with the three transducer types described above.. Relative cost is a comparison of the cost of a given scale to a no frills strain gage scale of the same load capacity.



PASSIVE STRAIN GAGE LOAD CELL
DISPLACEMENT TRANSDUCER
GEOMETRY CHANGES WITH LOAD

ACTIVE FORCE REBALANCE TRANSDUCER
INITIALLY SOFT BUT HAS THE SAME GEOMETRY
BEFORE AND AFTER BEFORE AND AFTER THE LOAD IS APPLIED
ELECTRICAL FEEDBACK PRODUCES A FUNCTIONALLY STIFF, LINEAR SYSTEM

Figure 35

Summary Table of Weight Scale Characteristics

Transducer Type	Load Range	Typical Rangeablility	Relative Cost	Comments
Strain Gage Load Cell	up to millions of lbs	1/2,000 to 1/5,000	1 to 2	- Least rugged - Insufficient sensitivity for laboratory scales - Nearly unlimited capacities and configurations available.
Capacitive Load Cell	Fractional to 50 lb	1/10,000 to 1/50,000	1.3 to 2.5	- Sensitive to shock - limited capacities available - high rangeability
Force Rebalance Load Cell	Micro gram to 13,000 lb	1/20,000,000 to 1/20,000	1.5 to 5.0	- Wide range of capacities available - highest rangeability - not suitable for dynamic measurements - Most optional features

9.4 Corner Loading Error When a scale manufacturer quotes the accuracy of his scale, usually he is referring to the accuracy when an object is placed on the platform so that its CG is in the center of the scale. If you place the object off center, then a moment will be created which tends to tip the platform of the scale. On many scales this will introduce an error. Depending on the internal mechanism in the scale, this error can be as large as 0.5%. High quality scales use parallel beam flexures and other compensating mechanisms, so that this effect can be as small as 0.001%. You can test the scale you are using by first placing a test weight in the center of the scale and measuring its weight. Then you move the weight to each of the corners of the scale and remeasure the weight. You may be surprised at how large the change is.

9.5 Weight vs Mass The mass of an object is fixed and is the same whether the object is on the earth or in outer space. Weight, on the other hand, is a force which depends on several factors which are related to the location of the scale. With the advent of force restoration technology, scales such as the Space Electronics YST Series have uncertainties in the order of 0.003% of full scale. With these scales, it is possible to measure an object at different locations of the earth and observe significantly different values for weight.

The force a mass exerts on a scale is affected by four factors:

1. The gravitational mass attraction to the earth at the particular location, which is in part related to the altitude

2. The gravitational mass attraction to the sun and moon at the particular location, which may reach 0.003% of the acceleration of earth gravity at certain dates during the year when the sun and moon align

3. The centrifugal force due to the rotation of the earth, which varies from zero at the north pole to a maximum value at the equator

4. The buoyancy of the object as it floats in a sea of air. This can be compensated for by determining the enclosed volume, and calculating the weight of the displaced air (whose density can vary due to the weather).

These factors combine to result in a change in the weight of an object of almost 1% over the surface of the earth, and about 0.2% over the contiguous USA! We put the exclamation point at the end of the sentence, because we frequently see specifications for weight accuracy of 0.02%, and none of these specifications mention the location on earth where this measurement is to take place. If a mass weighs 100 pounds at one location is the USA, it could weigh 99.8 lbs somewhere else in the USA.

To get around this problem, and contrary to popular beliefs, the world, including the USA, uses *Mass* rather than *Weight* to standardize and calibrate scales. To calibrate a scale, a standard calibration mass is placed on the scale. The scale is then adjusted until it reads the appropriate *standard weight*. The standard weight is the weight the mass would have at standard gravity (32.174 ft/sec^2).

Note: the traditional “Scales of Justice” balance beam compares one mass against another mass, and therefore the measurement does not vary with changes in gravitational field strength.

A problem occurs when a scale is calibrated at one location and then moved to another location to weigh an object. For large scale capacities, it is often not possible to bring a calibration weight to the new site, either because this weight is not available, or because of the problems of shipping a calibration mass weighing many thousands of pounds. Therefore, it is necessary to correct for the change in the acceleration of gravity between the site where the scale was calibrated, and the site where the object is being measured. The National Geodetic Information Center in Rockville, Md has data on the weight correction required for many locations on earth. If this data is not available, then another method is to determine the correction is to calibrate a small scale at the first site, and ship this scale to the new site, together with its calibration weight. The small calibration weight is then remeasured. If the measured value of this small weight is 0.05% high, then the acceleration of gravity is 0.05% higher at this new location, and the measurement of the large object must be divided by 1.0005 to correct for this change in the acceleration of gravity.

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ABOUT THE AUTHORS

Richard Boynton is President of Space Electronics, Inc., Berlin, Connecticut, a company he founded in 1959. Space Electronics, Inc. manufactures instruments to measure moment of inertia, center of gravity, and product of inertia. Mr Boynton holds a B.E. degree in Electrical Engineering from Yale University and has completed graduate studies in Mechanical Engineering at Yale and M.I.T. He is the author of over 67 papers, including 31 papers presented at SAWE conferences. He is the author of the SAWE Recommended Practice for Standard Coordinate Systems for Reporting the Mass Properties of Flight Vehicles. Mr. Boynton has been a member of SAWE for 30 years and is currently Director of the Boston Chapter. In 1992 he was elected a Fellow of the SAWE. He has designed many of the mass properties measuring instruments manufactured by Space Electronics.

Kurt Wiener is Assistant Chief Engineer of Space Electronics, Inc. Mr. Wiener holds a B.S. in Mechanical Engineering from M.I.T. and a M.S. in Industrial Education from Central CT State College. He is a former Associate Professor of Mechanical Technology at Vermont Technical College. Mr. Wiener is the principal instructor for the Mass Properties Seminars which are presented twice a year at Space Electronics. He has authored numerous papers for SAWE conferences.