Precise Measurement of Mass

by

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Abstract

Space vehicle weight is very critical. It is particularly important to have accurate weight if the vehicle weight is near the maximum allowable. Scales with accuracies as good as 0.002% are now available from companies such as Space Electronics. However, unless precaution is taken to correct for local gravity, air buoyancy, and a number of other factors, the high accuracy of the scale is not realized. In fact, considerations which are often overlooked can result in a total error of several percent. Some of the concerns are:

1. **Latitude** The acceleration of gravity varies from 9.780 m/s\(^2\) at the equator to 9.832 m/s\(^2\) at the poles (a difference of 0.53%). This is in part due to the centrifugal force resulting from the rotation of the earth, which varies from zero at the north pole to a maximum value at the equator, and in part due to the bulge of the earth at the equator, resulting in a greater distance to the mass center of the earth.

2. **Altitude** The gravitational mass attraction to the earth at a particular location varies as the square of the distance to the mass center of the earth, resulting in a variation by as much as 0.26% over the surface of the earth.

3. **Tidal effects** The gravitational mass attraction to the sun and moon at a particular location may exhibit a variation as large as 0.003% of the acceleration of earth gravity at certain dates during the year when the sun and moon align (this means that the scale could indicate a larger weight in the morning than at night).

4. **Gravity anomaly** Variations in the density of the earth’s crust in the vicinity of an object result in small variations in gravity at different locations. Nearby mountain ranges reduce the force of gravity on the object.

5. **Buoyancy** The object being measured displaces a volume of air, whose density can vary due to the weather. The resulting upward force is a function of object size. For large low density objects, failure to correct for buoyancy may result in errors of 0.5% or larger.

In addition, errors can be introduced by

6. **Moisture condensation or absorption of moisture** (evaporates in vacuum of space)
7. **Electrostatic attraction to the draft shield surrounding the scale**
8. **Magnetic attraction to nearby objects and to the earth’s magnetic north**
9. **Downdrafts or updrafts** (due to a temperature difference between object and ambient air)
Variations in the acceleration of gravity result in a change in the weight of an object of about 0.8% over the surface of the earth, and about 0.2% over the contiguous USA!

I put the exclamation point at the end of the sentence, because I frequently see specifications for a scale accuracy of 0.03% or better, yet these scales are used at latitudes that are significantly different from the latitude at which the scale was calibrated without any adjustment being made. Air buoyancy can cause a scale to read low by 0.5% or greater. Some of the other factors listed produce errors of 0.1% or more.

This paper discusses several issues:

1. How and when to correct for the acceleration of gravity when measuring mass.

2. How to correct for air buoyancy.

3. A very simple method to correct for both quantities automatically without knowing either the acceleration of gravity or the air density at a particular location.

In addition, this paper outlines the methods for determining local gravity, and summarizes the types of scales which are available and their accuracy. Finally, I have given my opinion on the subject of mass vs. weight, a topic which seems to result in endless confusion and disagreement.
1.0 Variations in the acceleration of gravity

1.1 Standard Weight  There is a common misconception that scales measure weight. In fact, most accurate scales measure mass. An object should “weigh” the same, no matter what scale it is weighed on, or where the scale is located. An ounce of gold must “weigh” one ounce in Miami or Boston. Otherwise you could buy the ounce of gold in Miami and sell it in Boston at a profit. If scales are used in commerce, an inspector will test them not for their accuracy in measuring force, but rather for their accuracy in measuring mass. In order to measure mass, some method must be used to compensate for the acceleration of gravity at the particular location in which a spring or load cell scale is used. This process is often called “calibrating the scale”. To calibrate a scale, a standard calibration mass is placed on the scale. The scale is then adjusted until it reads the appropriate standard weight. The standard weight is the weight the mass would have at standard gravity of 32.17405 ft/sec^2 (9.80665 m/sec^2).

A problem occurs when a load cell type scale is calibrated at one location and then moved to another location to weigh an object. For large scale capacities, it is often not possible to bring a calibration weight to the new site, either because this weight is not available, or because of the problems of shipping a calibration mass weighing many thousands of pounds. Therefore, it is necessary to correct for the change in the acceleration of gravity between the site where the scale was calibrated, and the site where the object is being measured.

1.2 Gravity Discussion
For two spheres of uniform density, the force of attraction is proportional to the square of the distance between the center of the spheres, as shown in figure 1.

The earth is not a perfect sphere, nor is its density uniform, so that there is a variation in attraction due to the irregularities of the surface and the non-uniform density. An object on the surface is attracted to every small particle in the earth. Those particles that are close to the object exert the strongest influence, since the force is inversely proportional to the square of the distance. Although the average of
the vector forces to each particle is approximately equal to a vector to the mass center of the earth, there will be some variation due to the concentration of dense materials in the earth’s crust at certain locations. There is a special science known as gravimetry which investigates variations in local gravity. The acceleration of gravity has been measured with an accuracy of better than 1 part in 10 million at many locations on earth.

If your object is located in a valley, there will be an attraction upward toward the peak of nearby mountains, and in fact there will also be a gravitational attraction sideways toward the mountain. These effects are of minor importance when measuring mass.

No one really knows what creates the force of gravity. Scientists have calculated that the attraction of gravity must act at least 10 billion times faster than the speed of light in order for the universe to be stable. There appears to be no limit to the extent of the attraction. Your body is being pulled outward from the earth by the attraction to the sun and the moon and every other object in the vast universe.

In "outer space" (a location in space so distant from any massive bodies such as stars that gravitational influence is negligible) the force of gravity approaches zero. Gravity is also zero at the mass center of the earth, since the attraction is equal in all directions. Shuttle astronauts in earth orbit experience free fall, not lack of gravity. Centrifugal force due to their rotation about the earth exactly counteracts the attraction of gravity, so that they remain at a fixed altitude.
The force of attraction of an object to the earth is defined by the law of mutual attraction given in Figure 3.

\[ F = \frac{G m_1 m_2}{r^2} \]

Figure 3 Mutual Gravitational attraction

where:

- \( G \) = universal gravitational constant, \( 6.67259 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \)
- \( m_1 \) = current accepted mass of earth in kg = \( 5.9736 \times 10^{24} \text{ kg} \)
- \( m_2 \) = mass of object in kg

Note: The currently accepted value of \( G \) given above has been obtained by experiment, and is therefore not known exactly.

\[ r = \text{radius from center of earth's mass in meters} = 6,378,100 \text{ m} \]

Note: (this is a nominal value; actual radius varies)

\[ m_1 = \text{current accepted mass of earth in kg} = 5.9736 \times 10^{24} \text{ kg} \]

This force can also be defined by a variation of Newton's second law of motion as given in Figure 4, where

\[ g = \text{acceleration of gravity in m/sec}^2 \]

Combining the equation of mutual attraction with Newton's second law yields an equation for the acceleration of gravity (figure 5)

\[ g = \frac{G m_1}{r^2} = \frac{3.985938 \times 10^{14}}{(6378100)^2} = 9.7982 \text{ m/sec}^2 \]

Example using nominal radius:

Note: This example is for the gravitational attraction only and doesn't include effect of centrifugal force due to earth's rotation.

1.3 Latitude Correction The most significant variable in determining the acceleration of gravity is the latitude. The value of \( g \) (shown in “gals” in fig. 6) is the smallest at the equator (due to the centrifugal force and the bulging of the earth).

The radius of the earth is approximately 22,000 meters more at the equator than at the north or south pole, due to bulge in earth which resulted from centrifugal forces while the earth was cooling. In addition, the centrifugal force due to the earth’s rotation counteracts the centripetal force due to the attraction of gravity.
The force is:

\[ F_{\text{cen}} = m_2 v^2 / R \]

where \( m_2 \) = mass of object being weighed

\( R \) = distance to earth’s rotation axis

\( v \) = speed (zero at poles; 463 m/s at equator)

The local value for \( g \) at sea level can be calculated using the formula

\[ g = 9.80613 \left( 1 - 0.0026325 \cos 2L \right) \]

where \( L \) is the latitude in degrees.

\( g \) is in m/sec\(^2\)

The traditional unit used by geologists for gravity is the “gal” (named after Galileo). 100 gals = 1 m/sec\(^2\). Therefore, the standard acceleration of gravity is 980.665 gals or 980665 milligals.

1.4 Altitude correction For locations on the surface of the earth, the gravitational attraction is inversely proportional to the square of the distance to the mass center of the earth. Therefore, gravity decreases as you increase altitude.

At sea level \( g = Gm_1 / r^2 \).

At an altitude \( H \), \( g_h = Gm_1 / (r+h)^2 = Gm_1 / (r^2 + 2rh + h^2) \)

\[ g/g_h = (r^2 + 2rh + h^2) / r^2 = 1 + 2h/r + h^2/r^2 \]

\( h^2/r^2 \) is extremely small, so \( g/g_h = 1 + 2h/r \)

\( g_h = g/(1+2h/r) \)

In addition, the centrifugal force increases as the radius increases, resulting in a further decrease in the acceleration of gravity. This centrifugal effect depends on the latitude. The decrease in \( g \) due to rotation is zero at the poles and reaches a maximum of about \( 1.567 \times 10^{-4} \) m/sec\(^2\) per km of altitude at the equator.
Formula to calculate reduction in measured weight due to increase in altitude

\[ g_h = g \left(1 - 3.92 \times 10^{-7} H\right) \]

where \( H \) = altitude in meters
\( g \) = gravity at sea level at the particular latitude
\( g_h \) = gravity at altitude \( H \) at the same latitude

Note: The above formula for altitude correction is based on the nominal radius of the earth and a latitude of 45 degrees. A more accurate calculation would be based on the actual radius and latitude at the specific location where the measurement was being made. However, the magnitude of the correction is small enough that this is not necessary.

The altitude correction is a relatively small number. If the altitude is increased by 3000 meters, then the measured weight decreases by 0.11%. However, high accuracy scales such as the Space Electronics Model YST series can detect changes in measured weight as small as that which results from an increase in elevation of only 40 meters, so the scale should be calibrated at the same altitude as the measurement.

1.5 Tidal variations An object on the surface of the earth is attracted to every celestial body. Most of these masses are too far away to have any significance on weight, but the sun and the moon do have a significance. If you have a scale whose accuracy is 0.003 % or better, you will notice that the weight of an object varies as a function of the time of day. This effect is most pronounced during spring and fall when the sun and moon align. This produces the “neap tides” that often cause flooding of marinas at these critical dates.

Figure 8 Measured weight varies slightly throughout the day due to changing attraction to the sun and moon
1.6 **Methods of measuring gravity** The process of measuring the acceleration of gravity has been refined to incredible precision. Absolute gravity can be measured to 1 part in 10 million and variation in gravity from one location to another to 1 part in 100 million. There are 4 ways in which this can be done:

1.6.1 **Free-fall technique**  Advances in technology have permitted absolute gravity measurements to be made by dropping a small object in a vacuum and measuring the rate of acceleration of the object as it falls in an evacuated chamber.

1.6.2 **Mass & Spring Technique**  This is a special kind of spring scale which has a very limited range of measurement and extremely high sensitivity. It uses a clever mechanism known as a “zero length spring”. This is a comparative instrument -- you null it out at one location and then measure the variation in static deflection as you move it to a new location.

1.6.3 **Pendulum method**  A simple pendulum has a time period which is proportional to the acceleration of gravity and the square of the length from pivot point to center of mass.

1.6.4 **Perturbation of orbiting artificial satellite**  Satellites are bound to the earth by the force of gravity. Otherwise they would fly off into space in a straight line. The height of a low earth orbit satellite will vary as it orbits the earth, due to the change in the force of gravity. This motion can be analyzed to create a map of gravity anomalies.

These methods are discussed in detail below.

1.7 **Free-fall Gravimeters**  Gravitational acceleration can be measured by dropping an object in a vacuum chamber and measuring speed as a function of time as the object accelerates. This is the method made famous by Galileo. He is supposed to have dropped a large and a small object from the leaning tower of Pisa and found that they both hit the ground at the same time. Subsequently he determined the formula given in figure 9.

Typical current free-fall laser interferometric gravimeters consist of three main pieces: an evacuated chamber with a free-falling test body, a reference test body (usually mounted on some type of isolation device), and a laser interferometer. Light is directed from a stabilized laser to a freely falling reflective test mass and also to a reference test mass held in isolation from non-gravitational accelerations and other background noise. The light is combined with the laser reference to produce optical interference fringes. The optical signal is

![Figure 9 Galileo performed the first free fall gravity experiment](image)
directed to a photo detector where the precise trajectory is sampled, resulting in many time and distance pairs. These data are then least squares fit by a computer to determine an absolute value for \( g \) with an accuracy as high as 1 part in 100 million. Absolute instruments provide independent information about the magnitude of the gravity field at a specified point, not the relative difference in gravity with respect to a known reference. Additionally, they do not suffer from tares, drifts, and calibration errors associated with relative instruments. On the negative side, instruments which use free fall methods are much more expensive than mass & spring gravimeters described below.

### 1.8 Mass & Spring Gravimeters
These instruments can measure relative gravity with a sensitivity of 1 part in 100 million. This type of device is much less expensive than the absolute gravimeter described previously. A clever spring mechanism is employed which is extremely soft. A small change in the acceleration of gravity causes a large change in the position of the mass. The operator views the position of the mass, and adjusts the spring mount position using an internal micrometer until the mass is leveled. A dial on the micrometer then indicates the value of \( g \). These instruments are sensitive to temperature change, so that temperature must be carefully controlled by a thermostatic heater. Early versions of this instrument were purely mechanical and contained no electronics.

### 1.9 Pendulum Method of Measuring Gravity
The earliest accurate measurements of the acceleration of gravity were made using a pendulum. A simple pendulum oscillates at a period which is inversely proportional to the square root of \( g \).

\[
T = 2\pi \sqrt{\frac{L}{g}}
\]

Solving for the acceleration of gravity =

\[
g = 4B^2 \frac{L}{T^2}
\]

Galileo was the first to use this method to determine the acceleration of gravity. The formula given above assumes that the amplitude of oscillation is small (i.e. less than 1 degree) so that \( A = \sin A \). Also oscillation must take place in a vacuum to avoid air damping and buoyancy errors. The diameter of the mass must be small relative to the length of the arm so that its MOI will have a negligible effect on oscillation period (or the effect can be mathematically compensated for). The pendulum must not twist in a torsional sense during oscillation, nor can the plane of oscillation vary. Temperature must be controlled to minimize thermal expansion so length \( L \) remains constant. One problem with this method is that the distance \( L \) is difficult to determine. If the pendulum was a round bob attached by a weightless arm, then \( L \) would be the exact distance from the pivot axis to the CG of the bob. In real life, \( L \) is an effective distance which is slightly
less than this, because the arm has finite mass and moment of inertia.

Ideal conditions can be approximated by suspending a round weight on two thin wires in a triangular bifilar configuration as shown below. The torsional and rocking motions are quickly damped, so that motion is in a single plane.

\[ F = mg \]

**Figure 12**

Bifilar Pendulum

**Figure 13**
2.0 Correcting for variations in the acceleration of gravity

There are four common methods to compensate for variations in gravity when measuring weight:

2.1 Method #1. By Re-calibrating the scale at a new location  The most accurate way of eliminating the effect of variation in the acceleration of gravity is to use a calibration weight whose mass has been determined by NIST. After setting up your scale at a particular location, you place the standard mass on the scale and adjust the readout of the scale to indicate the correct value. You have then corrected the scale to read standard weight. The standard weight is the weight the mass would have at standard gravity (32.17405 ft/sec\(^2\) or 9.80665 m/sec\(^2\)). Note: Once you have performed this calibration at a specific location, you have automatically corrected for the acceleration of gravity at the location. No mathematical manipulation of the data will be necessary to correct for latitude or longitude. However, for very precise measurements, you may still have to calibrate right before making a measurement to correct for the variation in gravitational attraction due to the sun and moon, since this varies with the time of day.

2.2 Method # 2. By using a small scale and calibration weight  If you have a means of calibrating your scale at one location, but you then need to move your scale to another location and you are unable to bring a large calibration mass to this location, you can accurately determine the correction necessary by the following procedure, which uses an additional small scale and calibration mass. Let’s say that you have a 10,000 pound capacity scale that has been calibrated at your factory in Seattle, using a primary standard set of weights that were adjusted at NIST. You need to bring this scale to Cape Canaveral to weigh a spacecraft, but you will not be able to bring the primary standard calibration weights. How do you correct the reading at Cape Canaveral?

2.2.1 You will need a precision small scale and a precision small calibration mass. For example you could use a one pound capacity scale and a one pound weight.

2.2.2 While you are still in Seattle, calibrate the small scale so that it reads exactly 1.000 pounds when the weight is placed on it. From this point on you may adjust the zero of this scale but not the span.

2.2.3 Bring the 10,000 pound scale, the small scale and the 1 pound weight to Cape Canaveral. When you arrive, place the 1 pound weight on the small scale and note the measured weight. If, in this example, the measured value of this small weight is 0.16% low (i.e. 0.9984 lb), then the acceleration of gravity is 0.16% lower at this new location, and the measured weight of the large object must be multiplied by 1.0016 to correct for this change in the acceleration of gravity. Note: This method does not correct for differences in air buoyancy at the two locations. See later discussion for more sophisticated method to correct for both gravity and buoyancy.
2.3 Method #3. By knowing the altitude and latitude of the location where the scale was calibrated and where the final measurement will take place. In some instances you will have no means of calibrating a scale. It may have been calibrated at the factory only. In this case you will need to know the acceleration of gravity at the location where the scale was calibrated and also at the location where you will be making your measurements.

Although anomalies in the earth’s crust will affect local gravity, gravity is mainly function of latitude and altitude. Gravity at a particular location can be calculated with a precision of about 0.002 % using the following formula:

\[
g = 9.80613 \left( 1 - 0.0026325 \cos 2L \right) \left( 1 - 3.92 \times 10^{-7} H \right)
\]

Where
- \(L\) is latitude in degrees
- \(H\) is altitude in meters
- \(g\) is expressed in m/s\(^2\):

The true mass can be calculated by the following formula:

\[
M = \frac{\left( g_C \right) (\text{MeasuredMass})}{\left( g_M \right)}
\]

where: \(g_C\) = acceleration of gravity where scale or load cell was calibrated
\(g_M\) = acceleration of gravity where measurement was made using scale or load cell

2.4 Method #4. By using information available from The National Geodetic Information Center. Values for the acceleration of gravity at specific locations can be purchased on CD ROM from the National Geophysical Data Center. The web address is [http://www.ngdc.noaa.gov/seg/fliers/se-0703.shtml](http://www.ngdc.noaa.gov/seg/fliers/se-0703.shtml). The price in 2001 was $350.

More information can be obtained from:

National Geophysical Data Center
NOAA EGC/1
325 Broadway
Boulder, CO 80303 USA
Phone: +1 303 497-6277
Fax: +1 303 497-6513
Email: seginfo@ngdc.noaa.gov

The true mass can be calculated by using the formula given in method #3.
3.0 Six effects which alter the measured weight at a fixed location

3.1 Buoyancy Large objects weigh less than small objects of equal mass, because of the buoyancy of an object as it floats in a sea of air. This effect is much more significant than most engineers realize. Buoyancy varies due to change in air density, which is affected by the weather and is a function of the atmospheric pressure, relative humidity and temperature. You can compensate for this effect by calculating the air density and determining the buoyancy effect for the volume of the object being weighed. This is discussed in more detail in the following section.

3.2 Tidal Variation As discussed previously, the gravitational mass attraction to the sun and moon varies during the year. It may reach 0.003% of the acceleration of earth gravity at certain dates during the year when the sun and moon align.

3.3 Condensation In areas with high humidity, sudden changes in temperature will produce condensation on the object being measured, which adds to the weight of the object. This most commonly occurs when an object is brought from an air conditioned space to a non air conditioned one. Condensation can be minimized by allowing an object’s temperature to equalize before weighing it.

3.4 Electrostatic attraction It’s remarkable how dramatic this effect can be if you are weighing a large lightweight object such as a mylar decoy reentry cone. If the object is covered by a clear plastic draft shield during measurement, the attraction between object and shield can be as much as 2% of the weight of the object.

3.5 Magnetic attraction If the object you are weighing contains permanent magnets, there will be an attraction to any magnetic material near or on the scale. There will also be a small attraction force to the magnetic north of the earth. Often an object has been magnetized by the magnetic chuck used in the machining of the object. It may be necessary to demagnetize the object before weighing it. Generally you can detect magnetic errors by repositioning the object on the weighing pan of the scale. If the readings are very sensitive to position on the pan, then the problem may be magnetic attraction (but it might also be the corner loading error of the scale). A Boy Scout compass is also a good way to test for magnetic attraction.

3.6 Drafts or air currents Generally, this effect will be quite obvious, since drafts will introduce a random variation in the readings. However, there are instances where drafts can produce a relatively steady downward or upward force. For example, if sunlight heats an object, then the updraft from the surface will produce an upward force, reducing the measured weight of the object. Conversely, if an object is brought in from an unheated storage area to be weighed, the cooler object will cause a downdraft, increasing the measured weight. These effects can be minimized by making sure that the object is at the same temperature as the surrounding air and by avoiding direct sunlight or bright lights.
4.0 Correction for Buoyancy

If an object is immersed in a fluid or gas, the object experiences a loss in weight equal to the weight of the fluid or gas it displaces. (Archimede’s Principal). This explains why a balloon rises. When an object is weighed in air, there is an upward force introduced which reduces the measured weight. For large lightweight objects, this effect can be quite significant. Air has an average density of 1.20 kg/m\(^3\). If an object has a volume of 1 cubic meter, then 1.2 kg must be added to the measured mass of the object to yield the true mass. This correction is particularly important for space vehicles whose mass is measured in the earth’s atmosphere.

To complicate this analysis, the scale was originally calibrated using a standard mass. This mass has volume and also experiences an upward force due to buoyancy. Therefore, for very accurate measurements, the calibration adjustment of the scale must be corrected for the buoyancy of this weight. This effect is very small. Usually a calibration mass is made of solid metal and has a density of 8000 kg/m\(^3\). Therefore, its buoyancy error is about 1 part in 6666.

4.1 Method to correct for buoyancy, using calculated air density

4.1.1 Calibrate the scale and adjust its output to read the exact value of the calibration weight.

4.1.2 Determine the density of the calibration weight (brass = 8570 kg/m\(^3\); stainless steel = 7870 kg/m\(^3\); cast iron = 7100 kg/m\(^3\))

4.1.3 Measure the weight of the object.

4.1.4 Determine the volume of the air displaced by the object (often not an easy task). Note: If openings allow air to enter a space, then this space is not part of the volume of air displaced.

4.1.5 Determine the approximate density of the object by dividing the measured weight by the volume of air displaced.

4.1.6 Determine the density of the air by using the following formula:

\[
d = \frac{0.020582h + 0.348444p - 0.00252t \ast h}{t + 273.15}
\]

where  
- \(p\) = atmospheric pressure in mbar (hPa) 
- \(h\) = relative humidity in % 
- \(t\) = temperature in degrees Celsius 
- \(d\) = air density in kg/m\(^3\)
4.1.7 Perform the buoyancy correction using the formula:

\[
\text{True Mass} = \text{Measured Mass} \times \frac{1 - \frac{d}{s}}{1 - \frac{d}{u}}
\]

where:
- \(s\) = density of calibration weight in kg/m\(^3\)
- \(u\) = density of unknown in kg/m\(^3\)
- \(d\) = density of air in kg/m\(^3\)

**Example**

Calculate air density where
- \(p\) = atmospheric pressure in mbar (hPa) = 1000 mbar
- \(h\) = relative humidity in % = 75%
- \(t\) = temperature in degrees Celsius = 23 °C

\[
d = \text{air density} = \frac{0.020582 \times 75 + 0.348444 \times 1000 - 0.00252 \times 23 \times 75}{23 + 273.15} = 1.167 \text{ kg/m}^3
\]

Calculate buoyancy correction where:
- \(s\) = density of calibration weight in kg/m\(^3\) = 8400 kg/m\(^3\)
- \(u\) = density of unknown in kg/m\(^3\) = 500 kg/m\(^3\)

\[
\text{buoyancy correction} = \frac{1 - \frac{1.167}{8400}}{1 - \frac{1.167}{500}} = 1.0022
\]

**TRUE MASS = MEASURED MASS \times 1.0022**

The measured weight will be 0.22% low if no correction is made for air buoyancy.
4.2 Method to correct for buoyancy by weighing two objects of identical mass but differing volume

There is an easier and more direct method of compensating for air buoyancy which does not require the measurement of atmospheric pressure, temperature, and relative humidity. The method requires two calibration weights of identical mass but differing volume. The difference in measured weight will be:

\[ W_1 - W_2 = d(V_2 - V_1) \]

Therefore:

\[ d = \frac{W_1 - W_2}{V_2 - V_1} \]

Since the volume difference \((V_2 - V_1)\) is precisely known, \(d\) can be calculated easily. The value of \(d\) is then used in the formula outlined in the previous section to determine the buoyancy correction.

The difficult task is to obtain two weights of identical mass but differing volume. These weights must be adjusted in a vacuum. Most scales cannot be operated in a vacuum, since there is no cooling for the electronics. A special type of remote operating beam balance is required. These weights are available from Space Electronics.

Figure 14 - Two calibration weights with identical mass and surface area but different volume are used to determine air buoyancy correction

Figure 15 The Space Electronics Model YST-CAL system consists of a scale with two weights of differing density which are used to determine the local acceleration of gravity and the air buoyancy correction
5.0 Instruments to Measure Mass

All scales make use of the acceleration of earth’s gravity to measure mass. This acceleration produces a downward force that must be counteracted in some way.

5.1 Direct mass measurement  Some scales, such as the old beam balance, measure mass directly by counteracting the downward force by another equal downward force on the opposite side of the pivot. On a beam balance, you add enough mass to the opposite pan to counterbalance the unknown mass. No force is measured. This type of scale automatically compensates for differences in the acceleration of gravity at different locations on earth. If you brought this type of scale to the moon, you would still get the same answer as on earth. Unfortunately, this type of scale does not lend itself to automatic operation and is rarely used any more.

5.2 Strain gage load cell scale  The most common scales still use strain gage load cells with typical accuracy of one part in 2000. These scales lend themselves to computer interfacing at relatively low cost. Pulsed DC power supplies and linearization circuits have allowed accuracies to one part in 5000 at higher cost.

5.3 Frequency shift technology scales  The next level of price with improved accuracy comes with new ceramic capacitive strain gages (for purposes of this paper, we will define accuracy as the ratio between uncertainty and load capacity). These scales can be made with accuracies up to one part in 30,000. Transducer stiffness is comparable with strain gage beam cells. These transducers have one drawback: they can be damaged if a hard object is dropped on the scale, since the ceramic spring is brittle and cannot withstand shock. Some newer scales using this technology incorporate spring shock dampers to minimize this problem.

5.4 Force restoration scales  The biggest innovation has been the application of force restoration technology to weight measurement. The newest generation of electronic force re-balance transducers can achieve accuracies on the order of one part in 10 million in laboratory balances. In the more common bench scale ranges up to 25 lb, the accuracy can approach one part in 1 million. In the larger sizes, 75 to 13,000 lb, accuracies are typically one part in 25,000 to one part in 50,000. At this time, the maximum load ratings of commercial scales available are on the order of 13,000 lb. Special devices have much larger capacities. Some high accuracy force measuring scales now are designed with a built-in calibration weight which compensates for the change in the acceleration of gravity. These scales are highly programmable to accommodate many weighing conditions. These include settings which vary: stability (i.e. animal weighing), parts counting on weight basis, selectable units of measurement, and they are fully compatible with computer interfacing. The disadvantages are price, and slower response time.
5.5 Summary Table of Weight Scale Characteristics  The table below compares scales with the three transducer types described above.  Relative cost is a comparison of the cost of a given scale to a no frills strain gage scale of the same load capacity.

<table>
<thead>
<tr>
<th>Transducer Type</th>
<th>Load Range</th>
<th>Typical Accuracy</th>
<th>Relative Cost</th>
<th>Comments</th>
</tr>
</thead>
</table>
| Strain Gage Load Cell         | up to millions of lbs       | 1/2,000 to 1/5,000 | 1 to 2        | - Least rugged  
- Insufficient sensitivity for laboratory scales  
- Nearly unlimited capacities and configurations available. |
| Capacitive Load Cell          | Fractional to 50 lb         | 1/10,000 to 1/30,000 | 1.3 to 2.5   | - Sensitive to shock  
- Limited capacities available  
- High accuracy |
| Force Restoration Technology  | Micro gram to 100,000 lb    | 1/20,000 to 1/20,000,000 | 1.5 to 5.0   | - Wide range of capacities available  
- Highest accuracy  
- Most optional features |

5.6 Force Restoration Weighing Principle
When an object is placed on the weighing pan, the pan deflects downward.  This pulls on the force link, causing the other end of the amplification beam to rise.  This produces an output from the position sensor, causing the current in the electromagnet to increase until the amplification beam is back to its original position.  A digital device determines the amount of current flowing through the electromagnet, which is proportional to weight.  An internal calculator translates the current magnitude into units of weight.
Since the position of the weighing pan and mechanical structure after the restoring force is applied is the same as the unloaded geometry, mechanical nonlinearity is eliminated. The transducer is inherently linear, like the time honored balance beam scale. In contrast, a strain gage load cell relies on the deformation of the sensitive spring element to generate an output, a process that is inherently nonlinear.

Another major advantage of this technology over strain gage load cells is that the output voltage of the transducer is in the order of 20 volts. Typical full scale output voltage of a strain gage load cell is 20 millivolts – resulting in a factor of 1000 less signal to noise ratio than a force restoration transducer.

The accuracy of force restoration weighing platforms is at least 10 times better than the best strain gage load cell scale. The high accuracy mass properties measuring instruments manufactured by Space Electronics for static CG and moment measurement use this force restoration technology.

<table>
<thead>
<tr>
<th>MODEL</th>
<th>Payload Capacity (lbs.)</th>
<th>Resolution (lbs.)</th>
<th>Typical Accuracy % of F.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>YST2.5FR</td>
<td>2.5</td>
<td>0.000,02</td>
<td>0.002%</td>
</tr>
<tr>
<td>YST25FR</td>
<td>25</td>
<td>0.000,2</td>
<td>0.002%</td>
</tr>
<tr>
<td>YST35FR</td>
<td>35</td>
<td>0.000,2</td>
<td>0.002%</td>
</tr>
<tr>
<td>YST330FR</td>
<td>330</td>
<td>0.002</td>
<td>0.004%</td>
</tr>
<tr>
<td>YST660FR</td>
<td>660</td>
<td>0.005</td>
<td>0.004%</td>
</tr>
<tr>
<td>YST1320FR</td>
<td>1,320</td>
<td>0.02</td>
<td>0.008%</td>
</tr>
<tr>
<td>YST2200FR</td>
<td>2,200</td>
<td>0.05</td>
<td>0.008%</td>
</tr>
<tr>
<td>YST6000FR</td>
<td>6,000</td>
<td>0.1</td>
<td>0.01%</td>
</tr>
</tbody>
</table>
Figure 17 Space Electronics YST Series weight platform being used to measure small rocket (shown with optional display pedestal and fixture)
Figure 18 - Weighing a missile prior to making mass properties measurements
6.0 Mass vs Weight (and English vs Metric)

In 1999, the Mars Climate Orbiter crashed as a result of a confusion over the system of units. The software program which controlled the thrusters was supplied with thrust data in pound-seconds but interpreted it as if it were newton-seconds, resulting in an underestimation of the thruster impulse by a factor of 4.45. This is only one of many thousands of errors that have occurred as a result of the confusion between Metric and English units.

Many of these errors are the result of a misunderstanding regarding the difference between mass and weight. If you place an object on a scale in Europe, you will read its mass (generally expressed in kg). However, if you place an object on a scale in the USA, you will read a value equal to the force exerted by the acceleration of gravity (generally expressed in lbf). Since you are really trying to use the scale to measure mass, when you weigh yourself on an American bathroom scale, it should read 6.22 slug rather than 200 lbf.

Traditionally, a dimensionally inconsistent correction factor is used to convert from one set of units to the other. The expression 1 kg = 2.205 lb is not valid. It is like comparing apples to oranges. Mass does not equal force. This traditional conversion factor is based on the value of standard gravity, which is 9.80665 m/sec².

Mass is related to weight through Newton's second law: \[ W = Mg \]

where \[ W = \text{the weight of the object (gravity force)} \]
\[ M = \text{the mass of the object} \]
\[ g = \text{the acceleration of gravity} \]

MASS is the QUANTITY OF MATTER in an object (its inertia), while WEIGHT is the FORCE that presses the object down on a scale due to the acceleration of gravity. The mass of an object is a fixed quantity; its weight varies as a function of the acceleration of gravity. The mass properties of an object are related to mass, not weight. Mass properties do not change as a space vehicle leaves the attraction of the earth and enters outer space.

If different names are used for weight and mass, then the problem of distinguishing between the two is minimized. The Metric SI system uses the word "Newton" for weight and the word "Kilogram" for mass. The Newton is defined as the force required to accelerate a 1 Kilogram mass by 1 meter per second². The aerospace industry has created a unit of mass called the "Slug." A one pound force is required to accelerate a one Slug mass at one ft/sec². If an object weighs 32.17405 lbf on earth, then its mass is one Slug.

Unfortunately, not all systems of units adequately differentiate between mass and weight. In the USA, the word "pound" is commonly used for both mass and weight, resulting in endless confusion and errors in calculating mass properties and dynamic response. Officially “pound” refers to mass (see, for example, NIST documents). However, the common usage of the word pound is the value you read on a scale, which is actually lbf. If the term "pound" is used to
describe a mass whose measured weight is one pound (force), this quantity MUST be divided by the acceleration of gravity in appropriate units to convert it to proper mass dimensions if it is to be used in mass properties calculations. Similarly, in metric countries the terms Kilogram and Gram are often, incorrectly, used to describe force as well as mass. To avoid confusion and uncertainty, an analysis of fundamental dimensions will confirm if correct units of measurement are being used and if conversion factors are being applied correctly to achieve desired results.

The various metric systems are fundamentally MASS, LENGTH, TIME systems with force being a defined or derived term. The U.S. systems are fundamentally FORCE, LENGTH, TIME systems with mass being defined or derived. Table One shows the three most commonly used systems of measurement. Time in seconds is used throughout.

DIMENSIONALLY CORRECT MEASURING SYSTEMS

<table>
<thead>
<tr>
<th></th>
<th>MASS</th>
<th>LENGTH</th>
<th>WEIGHT</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>SI (Metric)</td>
<td>Kg</td>
<td>Meter</td>
<td>Newton</td>
<td>9.80665 M/sec²</td>
</tr>
<tr>
<td>U.S. (inch)</td>
<td>Weight in lbf 386.0886 in.</td>
<td>Inch</td>
<td>Pound (lbf) 386.0886 in/sec²</td>
<td></td>
</tr>
<tr>
<td>U.S. (foot)</td>
<td>Slug</td>
<td>Foot</td>
<td>Pound</td>
<td>32.17405 ft/sec²</td>
</tr>
</tbody>
</table>

The U.S. inch system has no common name for the mass whose weight equals one pound, although this is sometimes called a “pound mass”. One pound mass is equal to one pound force divided by 386.088 inches/sec². Applying W = Mg shows that the system is dimensionally consistent.

\[
1 lb mass \times g = \frac{1 lb - sec^2}{386.09 \text{ in} \times sec^2} \times \frac{386.09 \text{ in}}{sec^2} = 1 lb weight
\]

The acceleration of gravity used to convert weight to mass is a fixed number which has been established as an international standard.

Editorial by Dick Boynton

The danger of using two sets of units was brought home dramatically by the failure of the Mars Climate Orbiter in 1999. I know that it would be incredibly expensive to convert the USA to the metric system. In fact, it could trigger an economic depression if it were required by law. However, there is one area where conversion could be relatively painless, and that is the conversion from the pound to the kilogram in the aerospace industry. Many industrial scales already read in both lbf and kg. The average person could easily multiply kilograms by 2 to get a rough estimate of the number of pounds.

Conversion to the kilogram would eliminate the endless confusion caused by a bogus unit (lbf) which has to be divided by the standard acceleration of gravity in order to be used in mass properties calculations.

Let’s get together and agree to eliminate the pound and use only kilograms in the aerospace industry.
7.0 Conversion factors  Most textbooks and handbooks do not provide sufficient precision in their conversion factors to be commensurate with the accuracy required in the measurement of space vehicles. The factors given below should be useful in this regard.

\[
g = 32.17405 \text{ ft/sec}^2 \text{ or } 9.80665 \text{ m/sec}^2
\]

**Dimensionally inconsistent conversion factors based on standard acceleration of gravity**

- 1 oz (ounce) = 28.349 52 gram
- 1 oz tr (troy ounce) = 31.103 48 gram
- 1 lbf (pound force) = 0.453 592 37 kg
- 1 kg = 2.204 622 6 lbf
- 1 kg = 9.806 65 Newtons
- 1 sh tn (short ton, US) = 907. 184 7 kg
- 1 ton (long ton, UK) = 1016. 047 kg
- 1 slug = 32.17405 lbf

**Force conversion factors**

- 1 dyne = 10^{-5} N
- 1 lbf (pound-force) = 4.448 22 N
- 1 kp (kilopond) = 9.806 65 N

**Mass conversion factors**

- 1 kg = 0.068 521 76 slug
- 1 t (tonne, metric) = 68.521 76 slug
- 1 slug = 14.593 904 kg
- 1 t (tonne, metric) = 1000 kg
8.0 Summary

Measured weight is affected by the following factors:

1. **Latitude** The acceleration of gravity varies from 9.780 m/s$^2$ at the equator to 9.832 m/s$^2$ at the poles (a difference of 0.53%).

2. **Altitude** causes a variation by as much as 0.26% over the surface of the earth.

3. **Buoyancy** The object being measured displaces a volume of air whose density can vary due to the weather. The resulting upward force is a function of object size. For large low density objects, failure to correct for buoyancy may result in errors of 0.5% or larger.

In addition, errors can be introduced by

4. **Moisture condensation on the object**
5. **Electrostatic attraction to the draft shield surrounding the scale**
6. **Magnetic attraction to nearby objects and to the earth’s magnetic north**
7. **Downdrafts due to a temperature difference between object and ambient air**
8. **Gravitational Tidal effects**

By virtue of the fact that standard masses are used to calibrate and adjust scales, **scales measure mass, not weight**. Metric scales indicate mass directly in kg; English scales display force in lbf; this number must be converted to mass by using the formula

\[ M = \frac{W}{g} \]

where \( g \) = Standard acceleration of gravity = 32.17405 ft/sec$^2$ or 9.80665 m/sec$^2$.

You must use the standard value of \( g \) when making the conversion, not the local acceleration of gravity at your location.

**Correcting for variations in the acceleration of gravity**

**Method #1. Re-calibrating the scale at a new location** Once you have performed a calibration using a mass which is traceable to NIST at a specific location, you have automatically corrected for the acceleration of gravity at the location. No mathematical manipulation of the data will be necessary to correct for latitude or longitude.

**Method #2. Correcting for the acceleration of gravity by using a small scale and calibration weight** If you have a means of calibrating your scale at one location, but you then need to move your scale to another location and you are unable to bring a large calibration mass to this location, you can accurately determine the correction necessary by the following procedure, which uses an additional small scale and small calibration mass.

1. Calibrate the small scale at the first location. **From this point on you may adjust the zero of this scale but not the span.**

2. When you arrive at the second location, place the small calibration weight on the small scale and note the measured weight. If the acceleration of gravity is lower at the new location, then the measured weight of the small calibration mass will be low.
3. Use your large scale to measure the unknown object. Then multiply the large scale reading by a correction factor equal to the true mass of the small calibration weight divided by the small scale reading in step 2.

Method #3. Correcting for acceleration of gravity by knowing the altitude and latitude of the location where the scale was calibrated and where the final measurement will take place. In some instances you will have no means of calibrating a scale. It may have been calibrated at the factory only. In this case you will need to know the acceleration of gravity at the location where the scale was calibrated and also at the location where you will be making your measurements.

Although anomalies in the earth’s crust will affect local gravity, gravity is mainly function of latitude and altitude. Gravity at a particular location can be calculated with a precision of about 0.002 \(^\circ\) using the following formula:

\[
g = 9.80613 \left( 1 - 0.0026325 \cos 2L \right) \left( 1 - 3.92 \times 10^{-7} H \right)
\]

Where:
- \(L\) is latitude in degrees
- \(H\) is altitude in meters
- \(g\) is expressed in \(\text{m/s}^2\):

The true mass, \(M\), can be calculated by the following formula:

\[
M = \frac{\left(g_C\right)(\text{MeasuredMass})}{\left(g_M\right)}
\]

where: \(g_C\) = acceleration of gravity where scale (or load cell) was calibrated
\(g_M\) = acceleration of gravity where measurement was made

Method #4. Correction for acceleration of gravity by using information available from The National Geodetic Information Center Values for the acceleration of gravity at specific locations can be purchased on CD ROM from the National Geophysical Data Center. The true mass can be calculated by using the formula given in method #3.

Method to correct for air buoyancy, using calculated air density

1. Calibrate the scale and adjust its output to read the exact value of the calibration weight.

2. Determine the density of the calibration weight (brass = 8570 kg/m\(^3\); stainless steel = 7870 kg/m\(^3\); cast iron = 7100 kg/m\(^3\))

3. Measure the weight of the object.

4. Determine the volume of the air displaced by the object.

5. Determine the approximate density of the object by dividing the measured weight by the volume of air displaced.
6. Determine the density of the air by using the following formula:

\[ d = \frac{0.020582h + 0.348444p - 0.00252t \times h}{t + 273.15} \]

where  
- \( p \) = atmospheric pressure in mbar (hPa)  
- \( h \) = relative humidity in %  
- \( t \) = temperature in degrees Celsius  
- \( d \) = air density in kg/m\(^3\)

7. Perform the buoyancy correction using the formula:

\[ \text{True Mass} = \text{Measured Mass} \times \frac{1 - \frac{d}{s}}{1 - \frac{d}{u}} \]

where  
- \( s \) = density of calibration weight in kg/m\(^3\)  
- \( u \) = density of unknown in kg/m\(^3\)  
- \( d \) = density of air in kg/m\(^3\)

**Method to correct for buoyancy by weighing two objects of identical mass but differing volume**  
Weigh each mass. The difference in measured weight will be: \( W_1 - W_2 = d(V_2 - V_1) \)

Therefore:

\[ d = \frac{W_1 - W_2}{V_2 - V_1} \]

Use the formula given in step 7 above.